HW 10.7 1-25 odd SKIP 21

#1 $\sum \frac{1}{n+3^n}$ $\frac{1}{n+3^n} < \frac{1}{3^n} \rightarrow$ GEOMETRIC $|r| = \frac{1}{3} < 1$

$\sum \frac{1}{n+3^n}$ CONVERGES BY COMPARISON TEST

#3 $\sum \frac{(-1)^n n}{n+2}$ $b_n = \frac{n}{n+2}$

b_n INCREASES

$\lim_{n\to\infty} \frac{n}{n+2} \neq 0 = 1$

$\lim_{n\to\infty} \frac{(-1)^n n}{n+2}$ = DOES NOT EXIST
OSCILLATES
(FROM 1 TO -1)

DIVERGES BY n^{th} TERM TEST

#5 $\sum \frac{n^2 2^{n-1}}{(-5)^n}$

$$\lim_{n \to \infty} \left| \frac{(n+1)^2 \cancel{2}^n}{(\cancel{-5})^{n+1}} \cdot \frac{(\cancel{-5})^n}{n^2 \cancel{2}^{n-1}} \right|$$

$$= \left| \frac{n^2+2n+1}{-5 n^2 \frac{1}{2}} \right| = \left| \frac{2\cancel{n^2}+2\cancel{n}^{0}+\cancel{1}^{0}}{-5\cancel{n^2}} \right|$$

$$= \frac{2}{5} < 1$$

converge

#13 $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

$$\lim_{n\to\infty} \left| \frac{3^{n+1}(n+1)^2}{(n+1)!} \cdot \frac{n!}{3^n n^2} \right| = \lim_{n\to\infty} \left| \frac{\cancel{3^n}\,3\,(n+1)^2 \cdot \cancel{n!}}{(n+1)\cancel{n!}\;\cancel{3^n}\,n^2} \right|$$

$$\cancel{\frac{3^n n^2}{n!}}$$

$$= \lim_{n\to\infty} \left| \frac{3}{n+1} \cdot \left(\frac{n+1}{n}\right)^2 \right|$$

$$\lim_{n\to\infty} \left| \frac{3}{n+1} \left(\frac{n+1}{n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \right)^2 \right| = \lim_{n\to\infty} \left| \underbrace{\frac{3}{n+1}}_{\to 0} \left(\frac{1+\overbrace{\tfrac{1}{n}}^{0}}{1} \right)^2 \right| = 0 < 1$$

CONVERGENT BY RATIO TEST

15 $\sum_{n=0}^{\infty} \frac{n!}{2\cdot5\cdot8\cdots(3n+2)}$

$$\lim_{n\to\infty}\left|\frac{(n+1)!}{2\cdot5\cdot8\cdots(3(n+1)+2)}\cdot\frac{2\cdot5\cdot8\cdots(3n+2)}{n!}\right|$$

$$\lim_{n\to\infty}\left|\frac{(n+1)\cdot\cancel{n!}}{\cancel{2}\cdot\cancel{5}\cdot\cancel{8}\cdots\cancel{(3n+2)}(3(n+1)+2)}\cdot\frac{\cancel{2}\cdot\cancel{5}\cdot\cancel{8}\cdots\cancel{(3n+2)}}{\cancel{n!}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(n+1)}{3n+3+2}\right| = \lim_{n\to\infty}\left|\frac{n+1}{3n+5}\right| = \frac{1}{3} < 1$$

$7! = 7\cdot6!$

CONVERGENT BY RATIO TEST

17) $\sum \underbrace{(-1)^n} 2^{1/n}$

$b = 2^{1/n}$ POS & DECREASING

$$\lim_{n\to\infty} 2^{1/n} = 2^0 = 1 \neq 0$$

THIS MEAN WE CAN'T USE THE A.S.T.

$$\lim_{n\to\infty} (-1)^n 2^{1/n} =$$

OSCILLATES

LIMIT DOES NOT EXIST

DIVERGES BY n^{th} TERM TEST

(19) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$ $b_n = \frac{\ln n}{\sqrt{n}}$ POSITIVE

THESE INCREASE

$\frac{\ln 1}{\sqrt{1}} + \frac{\ln 2}{\sqrt{2}} + \frac{\ln 3}{\sqrt{3}} + \frac{\ln 4}{\sqrt{4}} \ldots$ CAN'T TELL, FIND DERIVATIVE

THESE ~~XX~~ INCREASE

If $f = \frac{\ln n}{n^{1/2}}$ $f' = \frac{n^{1/2} \cdot \frac{1}{n} - \ln n \, \frac{1}{2} n^{-1/2}}{(n^{1/2})^2} = \frac{\frac{2}{2}\frac{1}{\sqrt{n}} - \frac{\ln n}{2\sqrt{n}}}{\frac{n}{1}} = \frac{2-\ln n}{2\sqrt{n}\, n}$

$= \frac{2 - \ln n}{n^{3/2}}$

$$2 - \ln n < 0$$
$$\underline{-2 \qquad -2}$$
$$\frac{-\ln n}{-1} < \frac{-2}{-1}$$

THIS SHOWS THE SERIES WILL EVENTUALLY BE DECREASING $\therefore$ $\ln n > 2$

$\leftarrow n > e^2$

FIND $\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \frac{\infty}{\infty} \overset{H}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2}n^{-1/2}} = \lim_{n\to\infty} \frac{1}{n} \cdot \frac{2\sqrt{n}}{1}$

$\frac{1}{2\sqrt{n}}$ $= \frac{2}{\sqrt{n}} = 0$

CONVERGES BY A.S.T.