10.4 #9

$$\sum \frac{\cos^2 n}{n^2+1} \qquad \frac{\cos^2 n}{n^2+1} \le \frac{1}{n^2+1} < \frac{1}{n^2}$$

CONVERGES

$\sum \frac{1}{n^2}$ CONVERGES (P-SERIES $P=2>1$)

SINCE $\frac{\cos^2 n}{n^2+1} < \frac{1}{n^2}$ BY COMPARISON TEST $\therefore$

$\sum \frac{\cos^2 n}{n^2+1}$ CONVERGES.

11) $\sum \frac{n-1}{n4^n}$ $\qquad \frac{n-1}{n4^n} < \frac{n}{n4^n} \leq \frac{1}{4^n}$ $\quad \left(\frac{1}{4}\right)^n$

$\sum \frac{1}{4^n}$ CONVERGES (GEOMETRIC w/ $|r| < 1$)

SINCE $\frac{n-1}{n4^n} < \frac{1}{4^n}$ BY COMP. TEST

$\sum \frac{n-1}{n4^n}$ MUST ALSO CONVERGE

(19) $\sum \frac{1+4^n}{1+3^n}$ $\quad \frac{1+4^n}{1+3^n} = a_n \quad b_n = \left(\frac{4}{3}\right)^n$ DIVERGES!

$$\lim_{n\to\infty} \frac{\frac{1+4^n}{1+3^n}}{\left(\frac{4}{3}\right)^n} = \lim_{n\to\infty} \frac{1+4^n}{1+3^n} \cdot \frac{3^n}{4^n} = \lim_{n\to\infty} \frac{1+4^n}{4^n} \cdot \frac{3^n}{1+3^n}$$

$$\lim_{n\to\infty} \frac{1+4^n}{4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} \cdot \frac{3^n}{1+3^n} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$\lim_{n\to\infty} \frac{\cancel{\frac{1}{4^n}}^{\,0}+1}{1} \cdot \frac{1}{\cancel{\frac{1}{3^n}}^{\,0}+1} = 1 > 0$$

BY LIMIT COMPARISON $\lim_{n\to\infty} \frac{1+4^n}{1+3^n}$ DIVERGES SINCE $\left(\frac{4}{3}\right)^n$ IS GEO. $|r|>1$.

(21) $\sum \frac{\sqrt{n+2}}{2n^2+n+1} = a_n$

$b_n = \frac{n^{1/2}}{\cancel{2}n^2} = \frac{1}{\cancel{2}n^{3/2}}$

P-SERIES

$p = 3/2 > 1$

CONVERGES

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n+2}}{2n^2+n+1}}{\frac{1}{n^{3/2}}}$$

$$\frac{1}{2}\sum \frac{1}{n^{3/2}}$$

$\sqrt{n^3}$

$$\lim_{n\to\infty} \frac{\sqrt{n+2}}{2n^2+n+1} \cdot \frac{n^{3/2}}{1} = \lim_{n\to\infty} \frac{\sqrt{n^4+2n^3}}{2n^2+n+1} \cdot \frac{\frac{1}{n^2}\left\{\frac{1}{\sqrt{n^4}}\right\}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{\sqrt{1+\frac{2}{n}}}{2+\frac{1}{n}+\frac{1}{n^2}}$$

$= \frac{1}{2} > 0$ BY LIM COMP TEST

a_n CONVERGES!