HW 10.3

$$\int \frac{1}{u^2+a^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

17) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4}$

$$\int_1^{\infty} \frac{1}{x^2+4}\,dx = \lim_{b\to\infty} \int_1^{b} \frac{1}{x^2+4}\,dx$$

$$= \lim_{b\to\infty} \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\Bigg|_1^b$$

$\downarrow$

P-SERIES

$p = 2 > 1$

CONVERGES

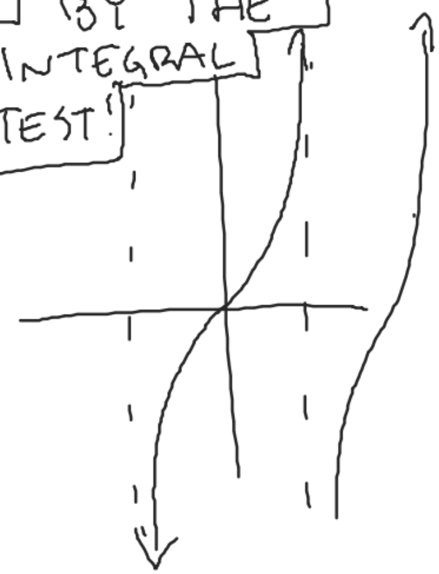
$$= \frac{1}{2}\lim_{b\to\infty}\left[\tan^{-1}\frac{b}{2} - \tan^{-1}\left(\frac{1}{2}\right)\right]$$

$$\frac{1}{n^2} \geq \frac{1}{n^2+4}$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)\right)$$

CONVERGES BY THE INTEGRAL TEST!

$$\tan = \frac{\sin}{\cos}$$



(19) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

$u = \ln x \quad v = \frac{x^{-2}}{-2}$

$du = \frac{dx}{x} \quad dv = x^{-3}\,dx$

$$\int_1^{\infty} \frac{\ln x}{x^3}\,dx = \lim_{b\to\infty} \int_1^b \ln x \cdot x^{-3}\,dx$$

$$= \lim_{b\to\infty}\left[\ln x \cdot \frac{x^{-2}}{-2} - \int_1^b \frac{x^{-2}}{-2} \cdot \frac{dx}{x} \right]$$

$$= -\frac{1}{2} \lim_{b\to\infty}\left[\frac{\ln x}{x^2}\Big|_1^b - \int_1^b \frac{dx}{x^3} \right]$$

$$= -\frac{1}{2} \lim_{b\to\infty}\left[\left(\frac{\ln b}{b^2} - \cancel{\frac{\ln 1}{1^2}}^{\,0} \right) - \frac{x^{-2}}{-2}\Big|_1^b \right]$$

$$= -\frac{1}{2} \cdot \lim_{b\to\infty}\left[\frac{\ln b}{b^2} - \left(\cancel{-\frac{1}{2b^2}}^{\,0} - \frac{1}{-2} \right) \right]$$

$$= -\frac{1}{2}\left[\lim_{b\to\infty} \frac{\ln b}{b^2} \overset{H}{=} \lim_{b\to\infty} \frac{\frac{1}{b}}{2b} = \lim_{b\to\infty} \cancel{\frac{1}{b \cdot 2b}}^{\,0} \right]\left(+ -\frac{1}{2} \right)$$

$\boxed{\frac{1}{4}}$ $\boxed{\text{CONVERGES}}$