Chapter 4 Review
AP Calculus BC
2017

Name: KEY

Hour: ________

You may use a calculator to begin with – you'll be asked to put it away later.

1. Find the limit: $\lim_{x\to 0} \frac{\sin x}{x^2 - x}$. $= \frac{0}{0} \overset{H}{=} \lim_{x\to 0} \frac{\cos x}{2x-1} = \frac{1}{-1} = \boxed{-1}$

FUTURE KILL

2. Find the limit: $\lim_{x\to 0} \frac{\tan x - x}{x^3}$. $= \frac{0-0}{0} = \frac{0}{0} \overset{H}{=} \lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2} = \frac{1-1}{0} = \frac{0}{0}$

$\overset{H}{=} \lim_{x\to 0} \frac{2\sec x \cdot \sec x \tan x}{6x} = \lim_{x\to 0} \frac{2\sec^2 x \tan x}{6x} = \frac{0}{0} = \frac{1}{3} \lim_{x\to 0} \sec^2 x \cdot \lim_{x\to 0} \frac{\tan x}{x}$

$= \frac{1}{3} \cdot 1 \cdot \lim_{x\to 0} \frac{\tan x}{x} = \frac{1}{3} \lim_{x\to 0} \frac{\tan x}{x} \overset{H}{=} \frac{1}{3} \lim_{x\to 0} \frac{\sec^2 x}{1} = \boxed{\frac{1}{3}}$

3. Find the limit: $\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$. $= \frac{\infty}{\infty} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{3}x^{-2/3}} = 3 \lim_{x\to\infty} x^{-1/3} = 3 \cdot \lim_{x\to\infty} \frac{1}{\sqrt[3]{x}} = \boxed{0}$

4. A formula for the derivative of f is given. How many critical values does f have?

$$f'(x) = 5e^{-0.1|x|} \sin x - 1.$$

SEE WHEN $f' = 0$ OR IS UNDEFINED

$\boxed{10}$ → COUNT THE X-INTERCEPTS!

USE: X_{min} -25
X_{max} 25
Y_{min} -1
Y_{max} 1

For problems 5 -7, use the function $y = \ln(3x^2+2x) + \cos x$ on the interval (0, 10). Round all answers to three decimal places.

5. What is the location of the absolute maximum?

$(6.577, 5.919)$ * FIND MAX OF F

USE
x min 0
x max 10
y min 2
y max 10

6. What are the locations of both local minimums?

$(2.219, 2.352)$
$(9.213, 4.632)$

* FIND MIN. OF F

7. What are the x-values of the inflection points?

* FIND 0'S OF $f'' = \frac{(3x^2+2x)(6) - (6x+2)^2}{(3x^2+2x)^2} - \cos x$

$x = 1.980, 4.630, 7.884$

8. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

$1200 = x^2 + 4xh$

$\frac{1200 - x^2}{4x} = h$ $\rightarrow$ $V = x^2 h$

$V = x^2\left(\frac{1200 - x^2}{4x}\right)$

$V = 300x - \frac{x^3}{4}$

$V' = 300 - \frac{3x^2}{4}$

$0 = 300 - \frac{3x^2}{4}$

$20 = x$

$V(20) = 300(20) - \frac{20^3}{4}$

$\boxed{V = 4000 \text{ cm}^3}$

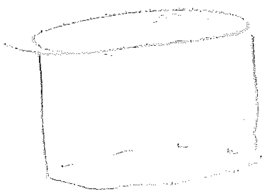
9. On what x value is the inflection point of the function $f(x) = \sqrt{3}x^3 + \sqrt{7}x^2 + 4x - 9$.

$f'(x) = 3\sqrt{3}x^2 + 2\sqrt{7}x + 4$

$f''(x) = 6\sqrt{3}x + 2\sqrt{7}$

$\boxed{\frac{-2\sqrt{7}}{6\sqrt{3}} \approx -0.509 = x}$

10. A fully-enclosed cylindrical container must hold 2L (2000 cm^3) of liquid. Find the dimensions of the container which will minimize the amount of material needed.



$V = \pi r^2 h = 2000$

$h = \frac{2000}{\pi r^2}$

$S = 2\pi r^2 + 2\pi r h$

$\longrightarrow S = 2\pi r^2 + 2\pi r \cdot \frac{2000}{\pi r^2}$

$S = 2\pi r^2 + 4000 r^{-1}$

$S' = 4\pi r - \frac{4000}{r^2}$

$0 = 4\pi r - \frac{4000}{r^2}$

$0 = 4\pi r^3 - 4000$

$\frac{1000}{\pi} = r^3$

$\sqrt[3]{\frac{1000}{\pi}} = r$

$\boxed{6.828 \text{ cm} \approx r}$

$\leftarrow$

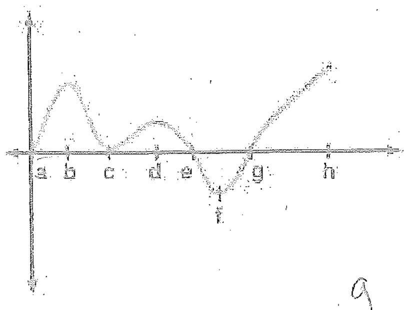
$h = \frac{2000}{\pi \sqrt[3]{\frac{1000}{\pi}}^{2}}$

$\boxed{h \approx 13.656 \text{ cm}}$

Now, PUT YOUR CALCULATOR AWAY! You won't get one for problems like these on the test, so you might as well try these out calculator-free. Let go. Use the Force.

11. Consider the function of f' given below on the interval $[a, h)$ where f is a particle in motion on a line. Please recognize that you are given a graph of the derivative of f, all the while kindly respecting the difference between soft and hard brackets.

Again, this is a graph of f', the derivative of f. Answer only on the interval [a, h).



a) f has a local min at: g

b) f has a local max at: e

c) f is concave up on the intervals a, b , c, d , f, h

d) f is decreasing fastest at: f

e) f' has an absolute min at: f

f) f' has an absolute max at: DNE

g) f has an absolute minimum value at: a.

h) $f(c)$ < $f(e)$. Fill in with $=, <, >$, or CBD (*Cannot be determined*).

Critical values are the x-values when the derivative is zero or undefined. Relative extrema (local minimums or local maximums) of a function only occur at critical values.

12. Find the critical value(s) on $f(x) = 3x^2 + 6x - 7$. Indentify each as a local max, local min, or DNE (does not exist). Show your work using the first derivative test.

$$f'(x) = 6x + 6 = 0$$
$$x = -1$$

	$6x+6$	f'	f
$x < -1$	$-$	$-$	$\downarrow$
$x > -1$	$+$	$+$	$\uparrow$

X = -1 IS A LOCAL MIN OF f, SINCE f' CHANGES FROM NEGATIVE TO POSITIVE AT X = -1

13. Find the critical value(s) on $f(x) = \ln(3x^2 + 12x)$.

$f' = \dfrac{6x+12}{3x^2+12x} = \dfrac{3(2x+4)}{3(x^2+4x)}$

$2x+4=0$

$\boxed{x=-2}$

$x^2+4x=0$

$x(x+4)=0$

$\boxed{x=0}$ $\boxed{x=-4}$

14. Find the value "c" on $[0, 9]$ that satisfies Mean Value Theorem for $f(x) = \sqrt{x} - \frac{1}{3}x$.

$m = \dfrac{f(9)-f(0)}{9-0} = \dfrac{\sqrt{9}-\frac{1}{3}(9)-0}{9} = 0$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3} = 0$

$\dfrac{1}{2\sqrt{x}} = \dfrac{1}{3}$

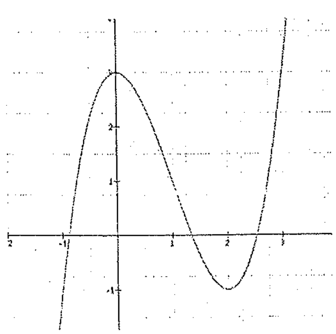
$2\sqrt{x} = 3$

$\sqrt{x} = 3/2$

$x = 9/4$

$\boxed{c = 9/4}$

15. The graph of f is given below (the scale is one in case you can't tell). Approximate the interval(s) upon which this function has $f'' \geq 0$:



$(1,3)$ or $(1,\infty)$

[concave up]

16. Find the intervals upon which f is increasing when $f'(x) = (x)(x+4)^2(x-\pi)$. Show your work using the first derivative test.   CRITICAL PTS: $x = 0$, $x = -4$, $x = \pi$

	x	$(x+4)^2$	$(x-\pi)$	f'	f
$x < -4$	$-$	$+$	$-$	$+$	$\uparrow$
$-4 < x < 0$	$-$	$+$	$-$	$+$	$\uparrow$
$0 < x < \pi$	$+$	$+$	$-$	$-$	$\downarrow$
$x > \pi$	$+$	$+$	$+$	$+$	$\uparrow$

f IS INCREASING ON $(-\infty, -4), (-4, 0), (\pi, \infty)$

17. On what interval(s) is the function $y = x^4 - 6x^3 + 12x^2 + 4x - 9$ concave down? Show your work using the second derivative test.

$y' = 4x^3 - 18x^2 + 24x + 4$

$y'' = 12x^2 - 36x + 24$

$0 = 12(x^2 - 3x + 2)$

$0 = (x-2)(x-1)$

$x = 2$, $x = 1$

POSSIBLE INFLECTION POINTS

	$(x-2)$	$(x-1)$	f''	f
$x < 1$	$-$	$-$	$+$	C UP
$1 < x < 2$	$-$	$+$	$-$	C DN
$x > 2$	$+$	$+$	$+$	C UP

f IS CONCAVE DOWN ON $(1, 2)$

(f'' IS NEGATIVE HERE!)

18. Find ALL of the critical values on $f(\theta) = 2\cos\theta + \sin^2\theta$.

$f'(\theta) = -2\sin\theta + 2\sin\theta \cdot \cos\theta$

$0 = 2\sin\theta(\cos\theta - 1)$

$0 = 2\sin\theta$

$0 = \sin\theta$

$\sin^{-1} 0 = \theta$

$\boxed{0 + \pi n = \theta}$

$0 = \cos\theta - 1$

$1 = \cos\theta$

$\cos^{-1} 1 = \theta$

$0 + 2\pi n = \theta$