Write each of the following functions as a composite of other functions:

Ex. $F(x) = \sin(3x^2)$

where

$f(x) = \sin(x) \quad g(x) = 3x^2$

Can be rewritten as

$F(x) = f(g(x))$

3.4: Chain Rule

If g is differentiable at x and f is differentiable at g(x), then $F(x) = f(g(x))$ and F' is given by:

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Example 1:

Find F'(x) if $F(x) = \sqrt{x^2 + 1}$

To begin, what is the "inside function" g(x)?

g(x)=

g'(x)=

f(x)=

f'(x)=

So F'(x)= f'(g(x))·g'(x)

$$= \frac{1}{2\sqrt{x^2+1}}(2x) = \frac{x}{\sqrt{x^2+1}}$$

Example 2:

Differentiate $y = \sin^2 x$

Remember that $\sin^2 x = (\sin x)^2$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)^2 = 2\sin x \cos x$$

All we did on Example 2 was combine the Power Rule and the Chain Rule!

so.......then couldn't we say:

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$

Ex. Find f'(x) if f(x) = $\frac{1}{\sqrt[3]{x^2+x+1}}$

First do this: f(x) = $\left(x^2+x+1\right)^{-\frac{1}{3}}$

f'(x) = $-\frac{1}{3}\left(x^2+x+1\right)^{-\frac{4}{3}}\frac{d}{dx}\left(x^2+x+1\right)$

= $-\frac{1}{3}\left(x^2+x+1\right)^{-\frac{4}{3}}(2x+1)$

Can we use the Chain Rule more than once?

For instance: find f' if $f(x)=e^{\sin(3x)}$

Begin by identifying the functions g,h,and j, such that $f(x)=g(h(j(x)))$.

$g(x)=$	$g'(x)=$
$h(x)=$	$h'(x)=$
$j(x)=$	$j'(x)=$

$f'(x) =$

Multiple Chain Rule use

Say you wanted to find f'(x) if f(x) = sin(cos(tan x))

$$f'(x) = \cos(\cos(\tan x)) \frac{d}{dx} \cos(\tan x)$$

$$= \cos(\cos(\tan x))[-\sin(\tan x)] \frac{d}{dx} \tan x$$

$$= \cos(\cos(\tan x))[-\sin(\tan x)](\sec^2 x)$$

Nasty but true...