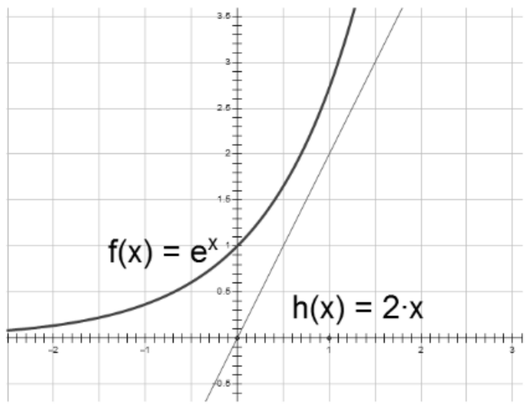
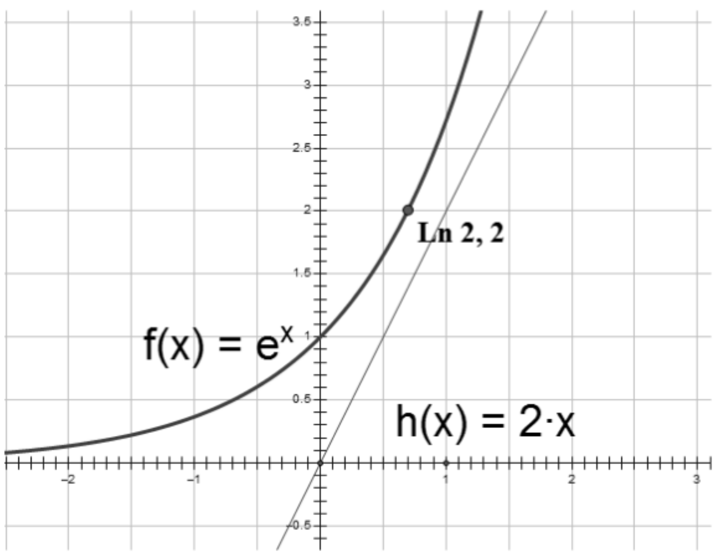
Do Now -





At what point on the curve $y = e^x$ is the tangent line parallel to the line $y = 2x$?

Since $y = e^x$, we know that $y' = e^x$.

Let the x-coordinate of the point we want be a. Then the slope of the tangent line at a is e^a.

This tangent line will be parallel to the line $y = 2x$ if it has the same slope, which is 2. So we need to find when $e^a = 2$. Solving for a, we get $a = \ln 2$.

Our point is $(\ln 2, 2)$, or about $(0.69, 2)$.

2.4 Product and Quotient Rules

Product Rule - 1st times d'2nd plus 2nd times d'1st

Quotient Rule - Bottom d'Top minus Top d'Bottom over Bottom Squared

Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

Ex. Differentiate

$$R(t) = \left(t + e^{t}\right)\left(3 - \sqrt{t}\right)$$

$$R'(t) = \left(t + e^{t}\right)\left(-\tfrac{1}{2}(t)^{\frac{-1}{2}}\right) + \left(3 - t^{\frac{1}{2}}\right)\left(1 + e^{t}\right)$$

$$R'(t) = \left(-\tfrac{1}{2}t^{\frac{1}{2}} - \tfrac{1}{2}(t)^{\frac{-1}{2}}e^{t}\right) + \left(3 + 3e^{t} - t^{\frac{1}{2}} - t^{\frac{1}{2}}e^{t}\right)$$

$$R'(t) = 3 + 3e^{t} - \tfrac{3}{2}t^{\frac{1}{2}} - t^{\frac{1}{2}}e^{t} - \tfrac{1}{2}(t)^{\frac{-1}{2}}e^{t}$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

Ex. Differentiate $y = \frac{x+1}{x^3+x-2}$

$$y' = \frac{(x^3+x-2)(1)-(x+1)(3x^2+1)}{(x^3+x-2)^2}$$

$$y' = \frac{x^3+x-2-3x^3-x-3x^2-1}{(x^3+x-2)^2}$$

$$y' = \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$$

Product Rule

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$