Ch 3.1

Derivatives of Polynomials and Exponential Functions

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad x^3 = f(x)$$

$$\lim_{h \to 0} \frac{(a+h)^3 - a^3}{h}$$

$$\lim_{h \to 0} \frac{\cancel{a^3} + 3a^2h + 3ah^2 + h^3 - \cancel{a^3}}{h}$$

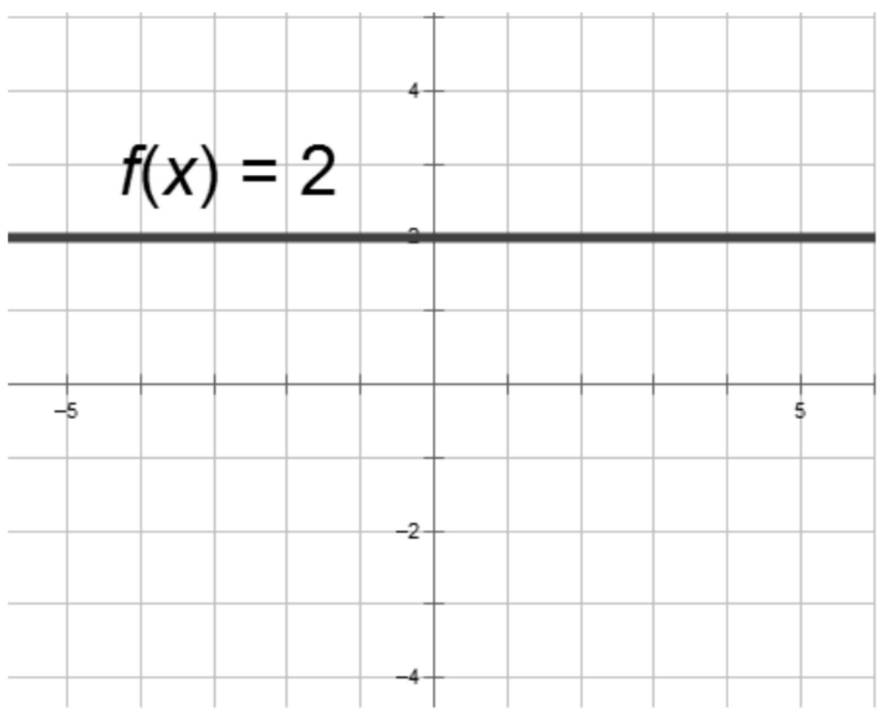
$$\lim_{h \to 0} \frac{\cancel{h}(3a^2 + 3ah + h^2)}{\cancel{h}} = 3a^2$$

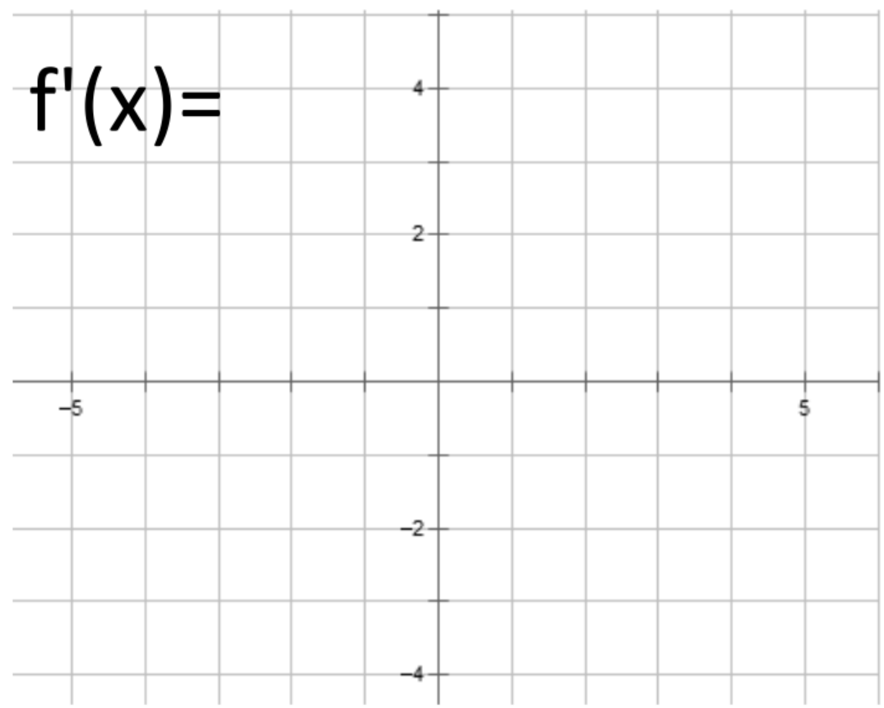
To begin:

Finding the derivative of a constant.

This isn't too bad if you know the definition of a derivative:

A derivative is the slope of a tangent line, or the limit of the slopes of the secant lines.





In other words:

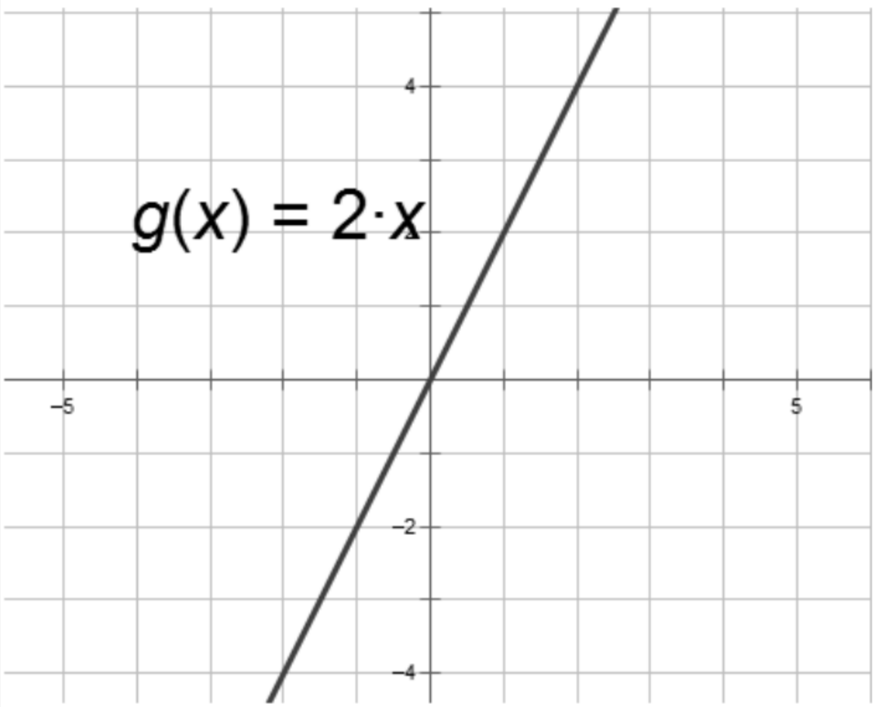
$$\frac{d}{dx}(c) = 0$$

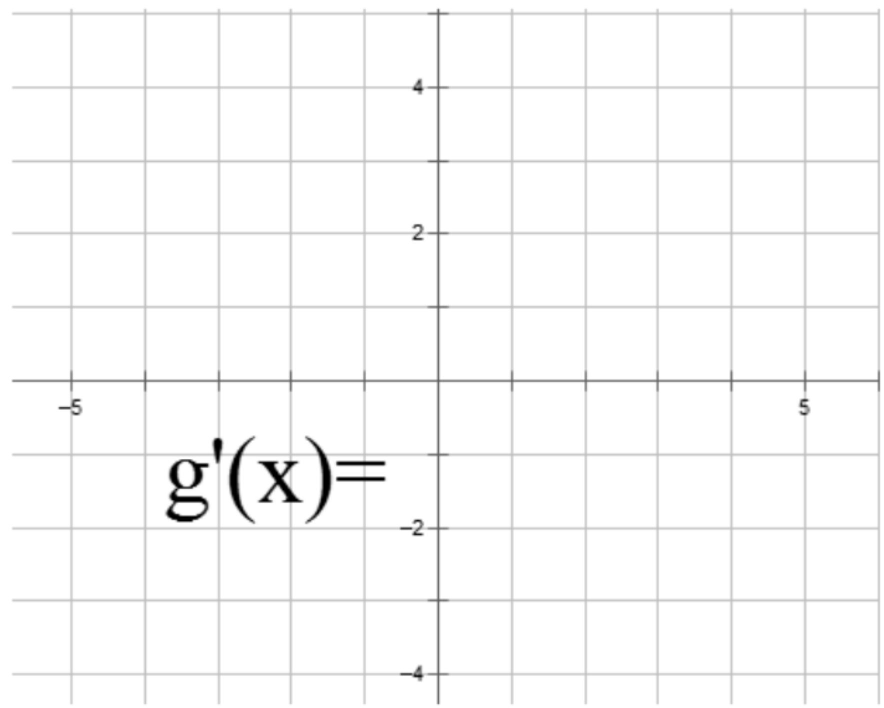
whenever c is a constant value.
The slope of all of these tangent lines is 0!!

Okay, that was easy.
What about another straight line

(courtesy of the Dept. of Redundancy Department)

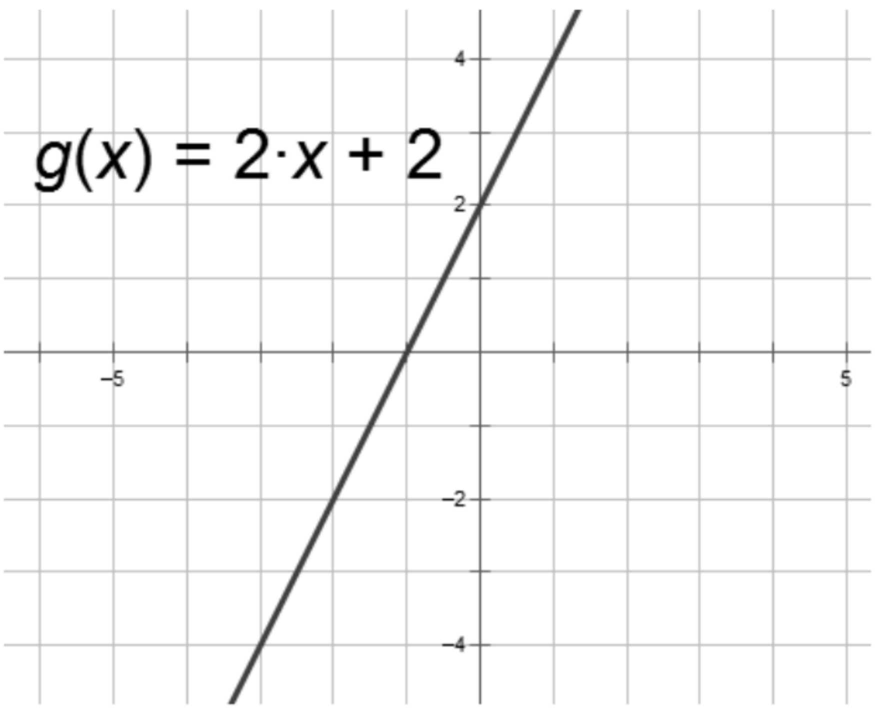
like f(x)=2x

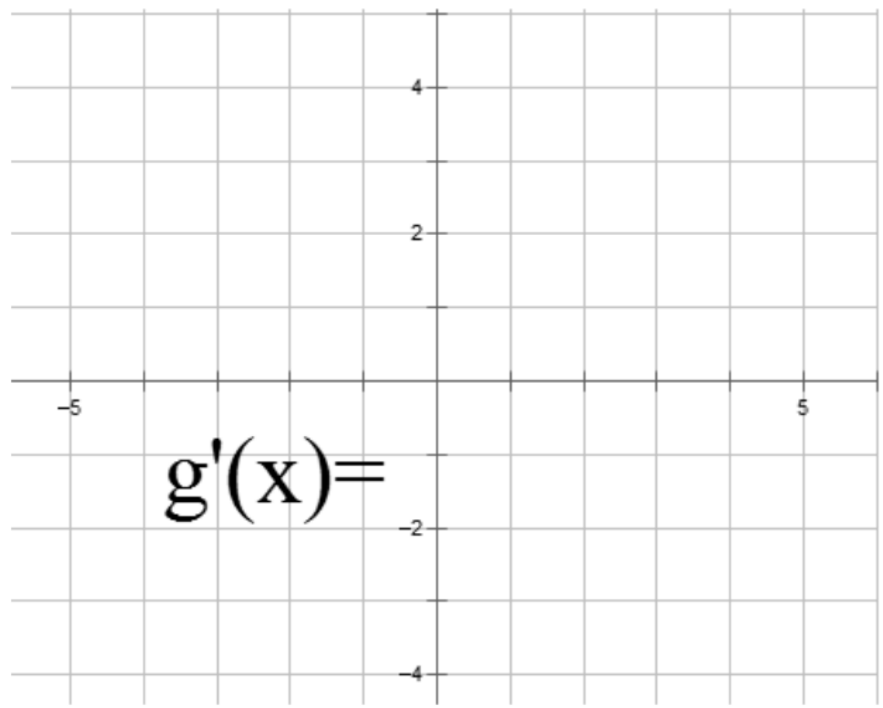




What if the function is slightly "tweaked" like the one below?

Remember that the derivative of a constant is ____________

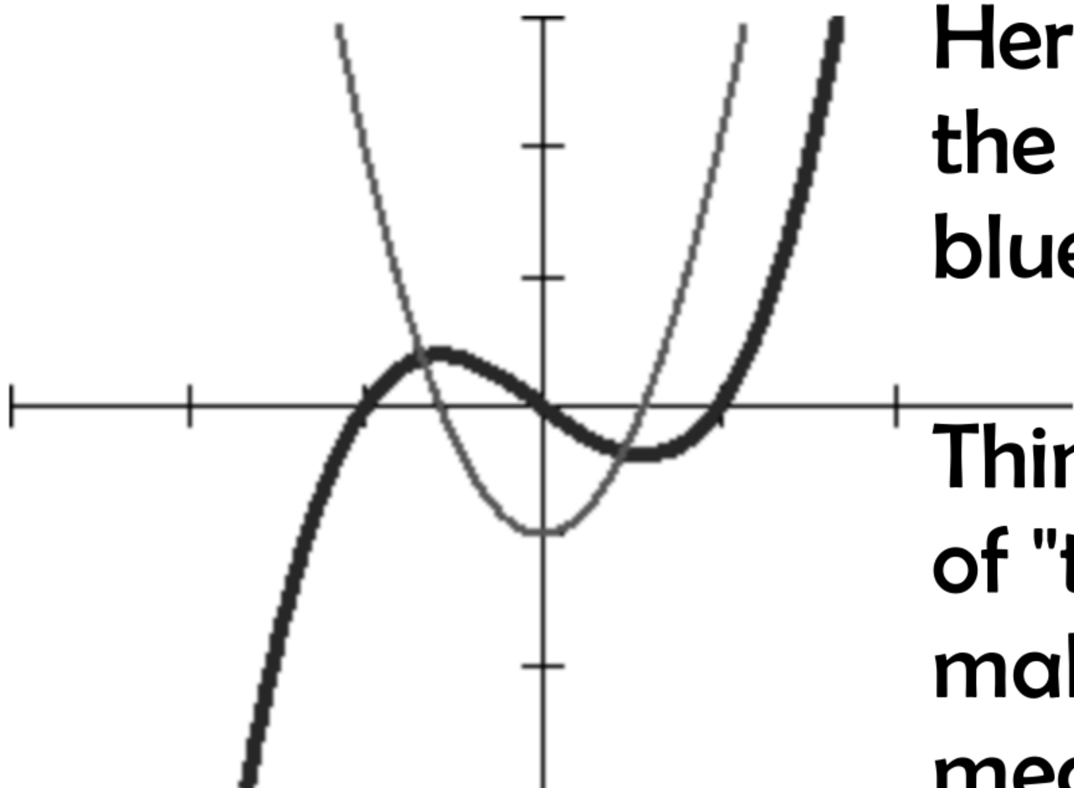




Derivative of a Linear Function:
if $f(x) = mx+b$
then $f'(x) = m$

Hey, so if $f'(x)$ is a constant, then what is f'' of the function f?

Check this out - remember graphing a function and it's derivative, and the derivative is a degree lower than the original? No, you don't remember? OK, take a look:



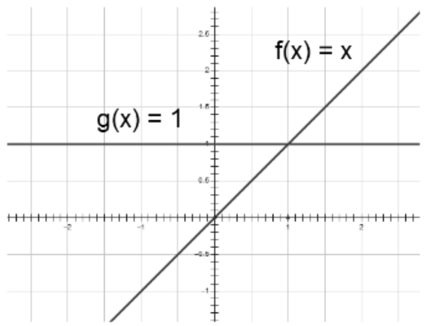
Here the red curve is the derivative of the blue function.

Think about the amount of "turns" a function can make, and what a "turn" means to the derivative.

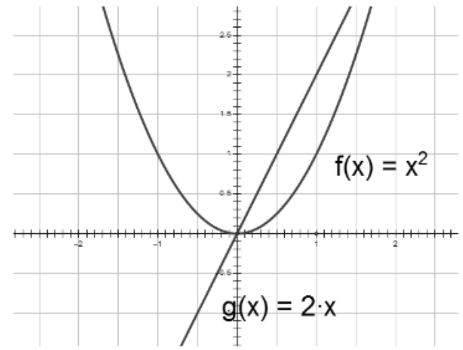
In any case, f(x) is cubic, and f'(x) is quadratic.

See if you can find the pattern; most of these we've already done:

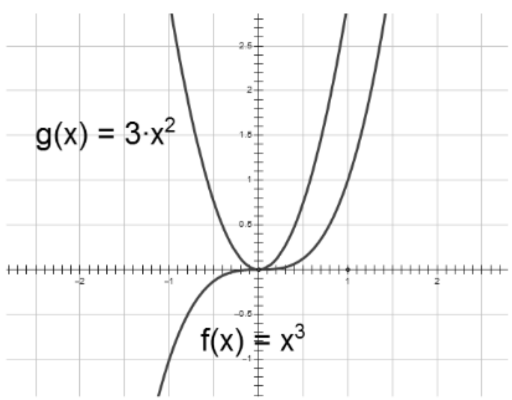
$$\frac{d}{dx}x = 1$$



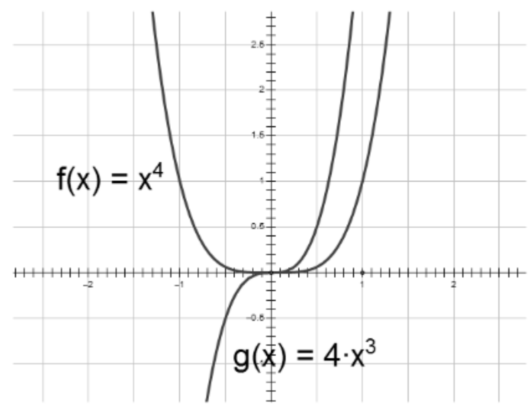
$$\frac{d}{dx}x^2 = 2x$$



$$\frac{d}{dx}x^3 = 3x^2$$



$$\frac{d}{dx}x^4 = 4x^3$$



Here's where every Calc student gets ticked off.

Rather than do this: $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

You can , if n is a real number, do this:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

This is called the **Power Rule**

So say you wanted to differentiate (find the derivative of)

$$f(x) = \sqrt[3]{x^2}$$

$$f'\left(x\right) = \frac{d}{dx}\sqrt[3]{x^2} = \frac{d}{dx}x^{2/3} = \frac{2}{3}x^{-1/3}$$

See how the new exponent is n-1?

Here's a sweet trick using the Constant Multiple Rule

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Ex. Find $\frac{d}{dx} -5x^3$

$$= \frac{d}{dx}(-5)x^3$$
$$= -5\frac{d}{dx}x^3$$
$$= -5 \cdot 3x^2 = -15x^2$$

A couple more rules just say that you find each differentiable function separately if you're adding or subtracting them together.

The Sum Rule

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule

$$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Finally, as we saw on that last assignment, the derivative of e^x is e^x.

The Derivative of the Natural Exponential Function

$$\frac{d}{dx}e^{x}=e^{x}$$

Normal Line - the line that goes through the same point as the tangent line, but with a PERPEDICULAR slope to the tangent line.

Oh, and one more thing:

The derivative of position is velocity,

and the derivative of velocity is acceleration.

So, the SECOND derivative of position is acceleration. Don't ever forget this. Ever.