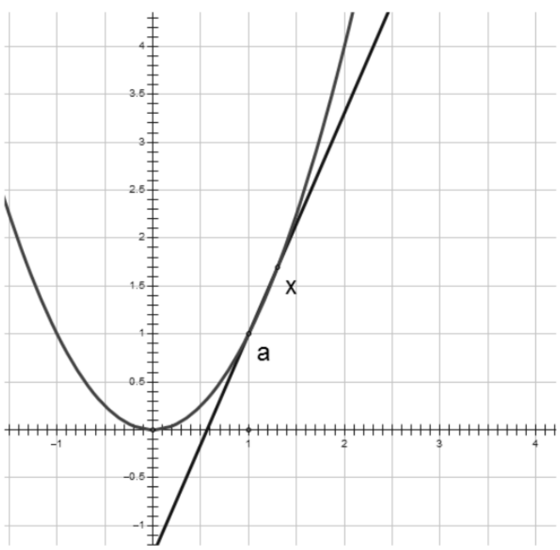
2.2 - The Derivative as a Function

To begin, recall that we defined the derivative of f as:

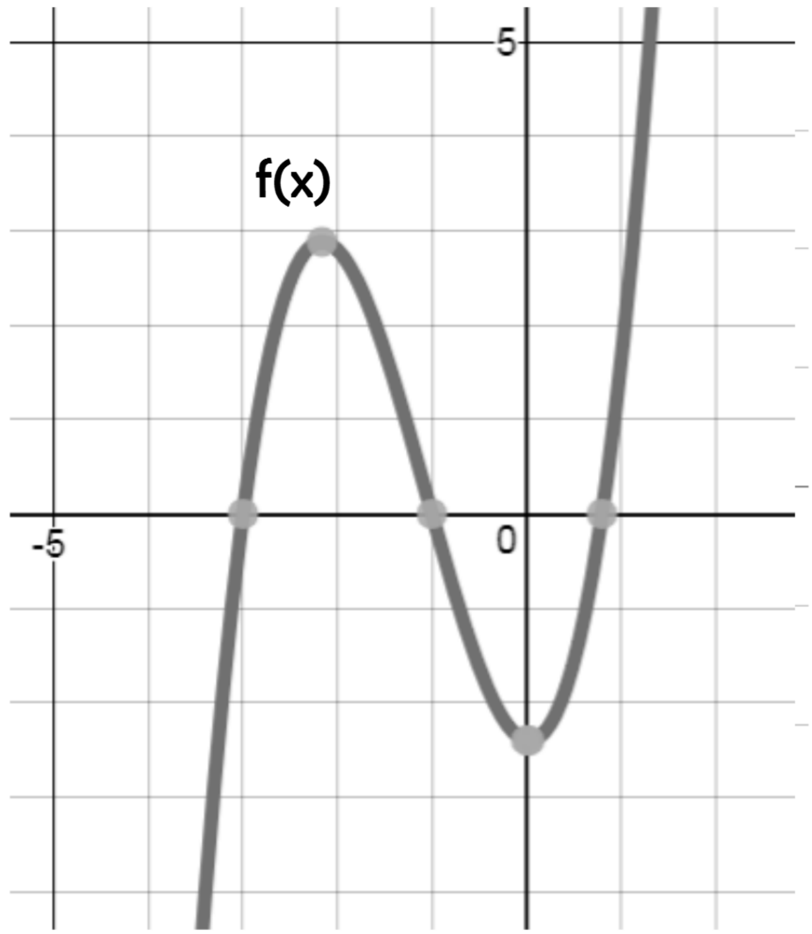
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

...That is, f'(x) is the function that graphs the derivative, or slope of the tangent lines, of f(x). We use limits because we are interested in when the two points on this secant line, a and x, approach each other. "h" denotes the distance between a and x.



Order the following values from least to greatest:

f'(-2.5) f'(-2) f'(-1) f'(0) f'(1)

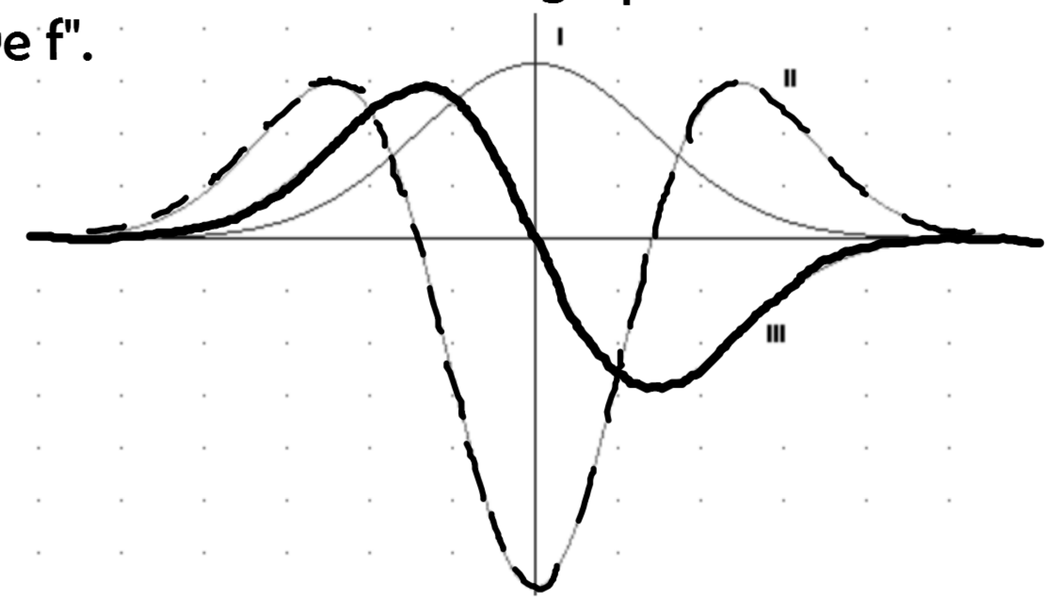


The graphs of function f, its first f ' and second derivatives f'', are shown below. Identify the graph of function f, the graph of its first derivative f ' and the graph of its second derivative f''.

f(x)

f'(x)

f''(x)



Definition:
A function f is **differentiable** at a if f'(a) exists. It is **differentiable** on the open interval (a,b) if it is differentiable at every number in the interval.

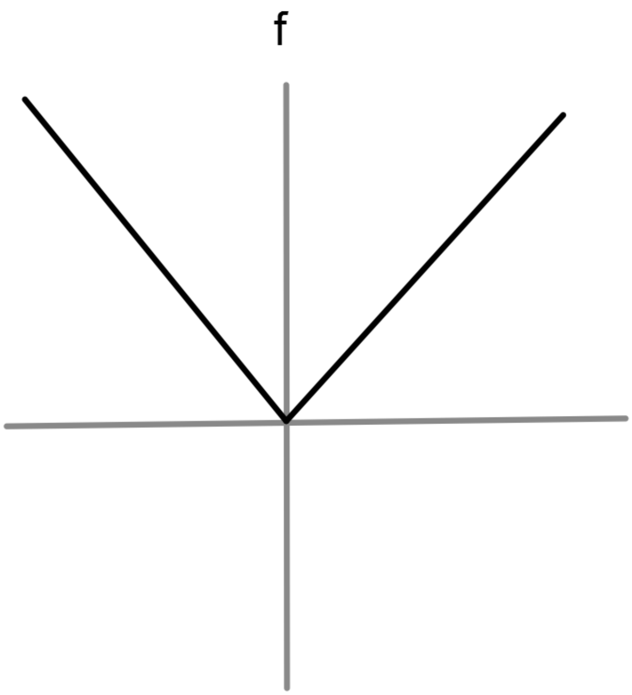
What does it mean to say that f'(a) exists? What must be true?

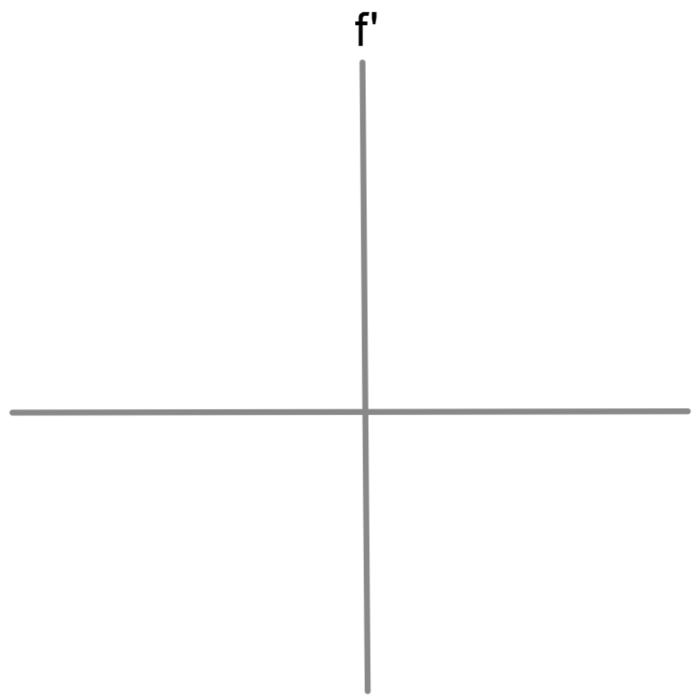
Theorem:
If f is differentiable at a, then f is continuous at a.

New Piece of Vocabulary!!! (for some of you)
Converse: If A then B ------> If B then A.

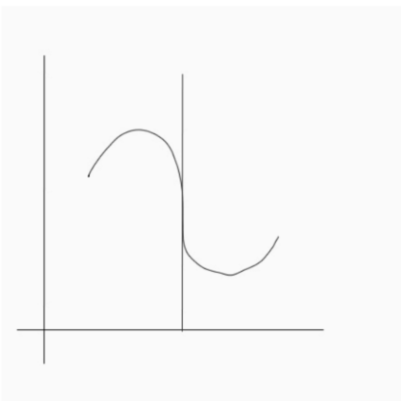
What is the converse of:
If f is differentiable at a, then f is continuous at a.

Example:
Determine whether $f(x)=|x|$ is differentiable on all real numbers.

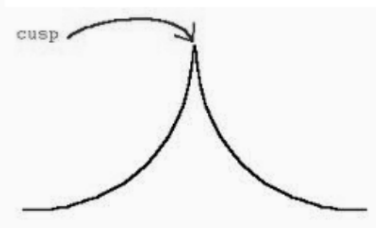




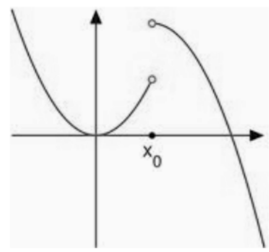
Three reasons a function fails to be differentiable:



vertical tangent



cusp or corner



discontinuity

Notation:

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x)$$

Liebniz's notations: (Gottfried Wilhelm Liebniz)
Controversial inventor of calculus....along with.....????

1. Show that there exists a solution to ln x + 3x = 4 on [1, 3].

2. Is it possible for $\lim_{x \to 4} f(x) = 5$ and $f(4) = 3$? Explain.

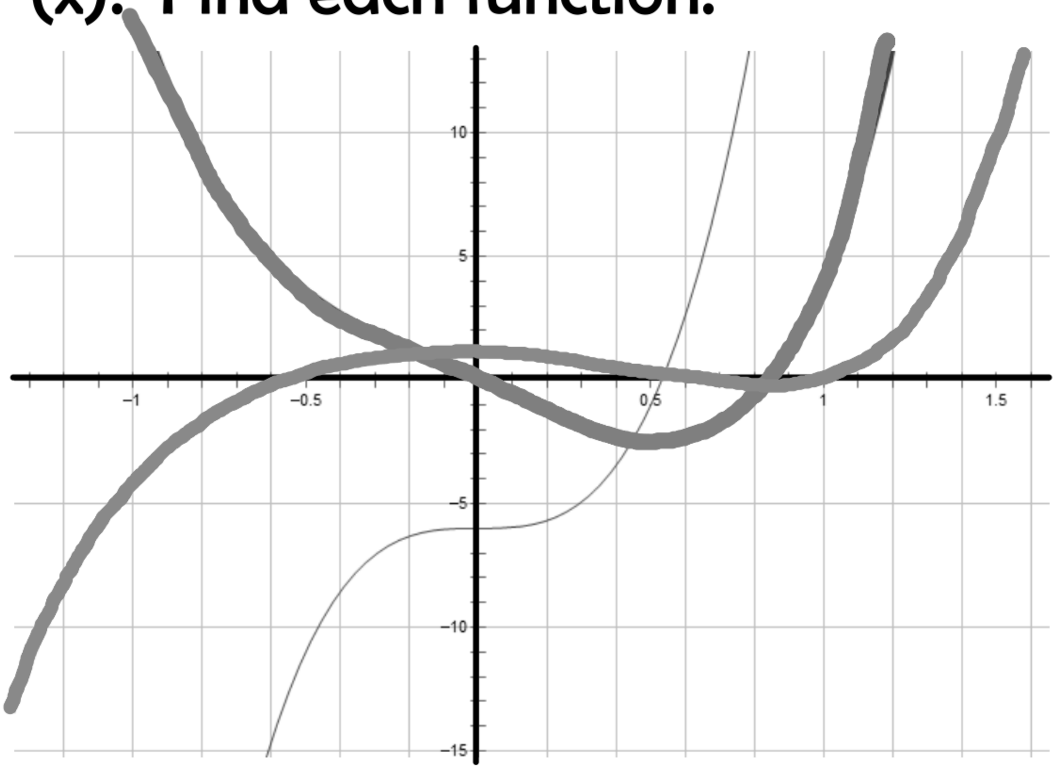
3a. Find the derivative of the function below by using the definition of a derivative.

$$f(x) = 13 - 4x + 7x^2$$

b. Find f' (-1)

c. Write the equation of the tangent line to f(x) at x = -1

Below are functions that represent f(x), f'(x), and f''(x). Find each function.



f(x)

f'(x)

f''(x)

Example: if $f(x)=x^3-x$, find a formula for $f'(x)$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\left[(x+h)^3-\left(x+h\right)\right]-\left(x^3-x\right)}{h}$$

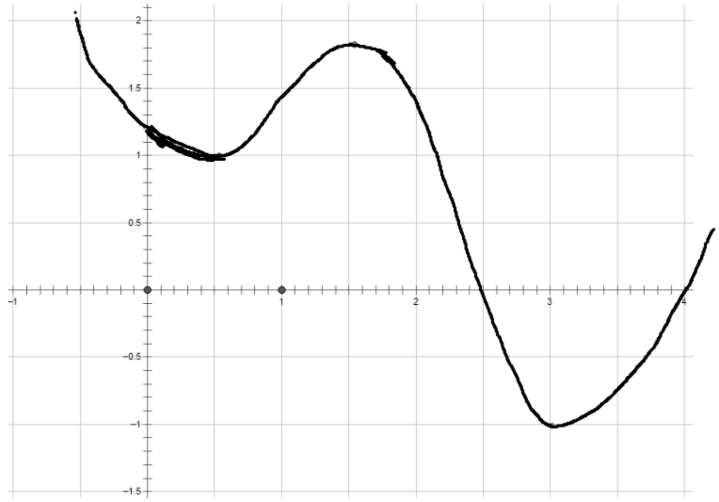
$$=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x-h-x^3+x}{h}$$

$$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3-h}{h}$$

$$=\lim_{h\to 0}\left(3x^2+3xh+h^2-1\right)=\boxed{3x^2-1=f'(x)}$$

Using the definition of f', sketch f' given the graph of f(x).

f(x)



f'(x)