Do Now - Sept. 11, 2017

What is...

$$\lim_{t \to -\infty} \frac{t-6}{t^2-6t}$$

2.6 Limits at Infinity: horizontal asymptotes.

A horizontal asymptote, L, exists if, as the function goes to positive or negative infinity, f(x) can be made arbitrarily close to L by taking x as a sufficiently large positive or negative value.

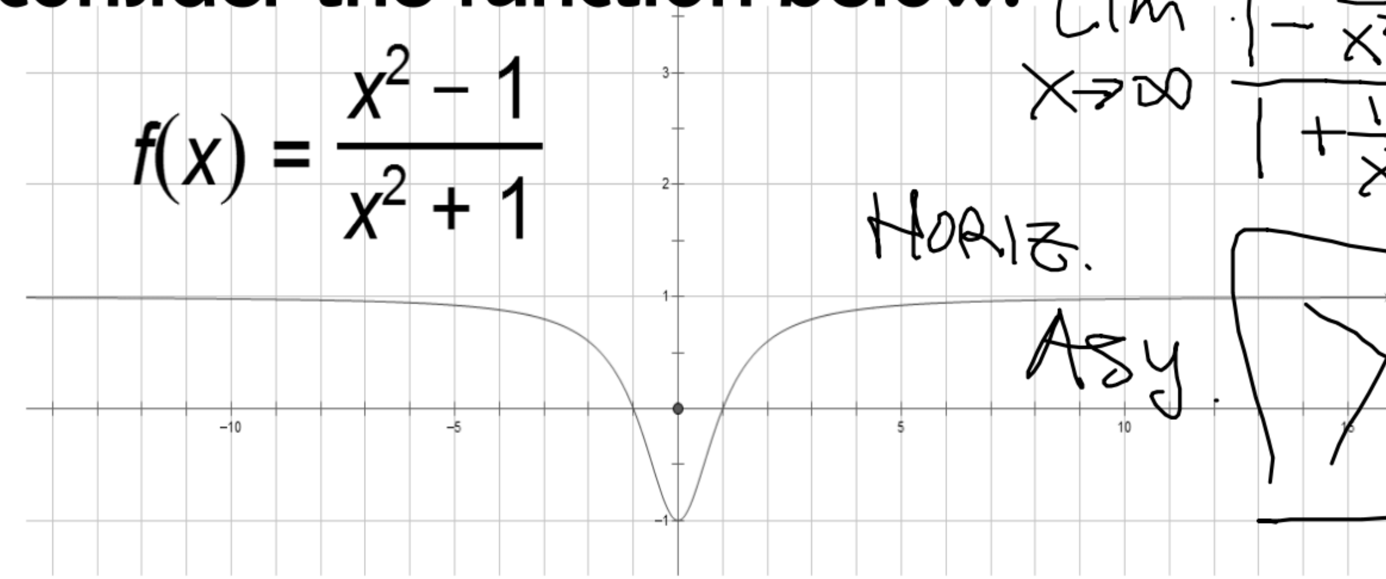
Key idea:

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

Example:

consider the function below.

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$



$$\lim_{x \to \infty} \frac{x^2 - 1}{x^2 + 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$\lim_{x \to \infty} \frac{1 - \frac{1}{x^2}}{1 + \frac{1}{x^2}} = \frac{1 - 0}{1 + 0}$$

HORIZ. Asy. $\boxed{y = 1}$

First off, where does it seem like there exists a horizontal asymptote?

Example:

Recall that the limit as x approaches infinity, f(x)=1/x goes to 0.

Evaluate:

$\frac{x}{x^2} = \frac{1}{x}$

Method to solve:
Multiply by clever form of one.
Use 1 over highest degree of denominator

$$\lim_{x \to \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$\lim_{x \to \infty} \frac{3 - \frac{1}{x} - \frac{2}{x^2}}{5 + \frac{4}{x} + \frac{1}{x^2}} = \boxed{\frac{3}{5}}$$

Do Now - 9/12/17

1. Draw a function that is defined on a closed interval (so, the domain is [a, b] - you choose a and b) that FAILS the Intermediate Value Theorem.

2. Graph, according to the following conditions:

$$f(0) = 3 \qquad \lim_{x \to 0^-} f(x) = 4 \qquad \lim_{x \to 0^+} f(x) = 2$$

$$\lim_{x \to -\infty} f(x) = -\infty \qquad \lim_{x \to 4^-} f(x) = -\infty$$

$$\lim_{x \to \infty} f(x) = 3 \qquad \lim_{x \to 4^+} f(x) = \infty$$

Most limit laws hold true...But wait.

Evaluate: $\lim_{x\to\infty}(x^2 - x) = \infty^2 - \infty = \infty$

$\boxed{\text{DON'T}}$

$$\lim_{x\to\infty} x^2\left(1 - \frac{1}{x}\right)$$

$$\lim_{x\to\infty} x^2 \cdot \lim_{x\to\infty}\left(1 - \frac{1}{x}\right)$$

$$\infty \cdot 1 = \boxed{\infty}$$

$x^2 - x$

$x(x-1)$

Look at this ugly thing:

$$\lim_{x\to\infty}\sqrt{x^2+1}-x=\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$$

$$=\lim_{x\to\infty}\frac{\left(x^2+1\right)-x^2}{\sqrt{x^2+1}+x}=\lim_{x\to\infty}\frac{1}{\sqrt{x^2+1}+x}$$

At this point we want to divide the numerator and denominator by 1/x (x being the highest power in the denominator, right?)

$$=\lim_{x\to\infty}\frac{1}{\sqrt{x^2+1}+x}\frac{\frac{1}{x}}{\frac{1}{x}}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{\sqrt{x^2+1}+x}{x}}$$

$5\sqrt{x+2}$

$\sqrt{5}\sqrt{x+2}$

$\sqrt{5(x+2)}=$

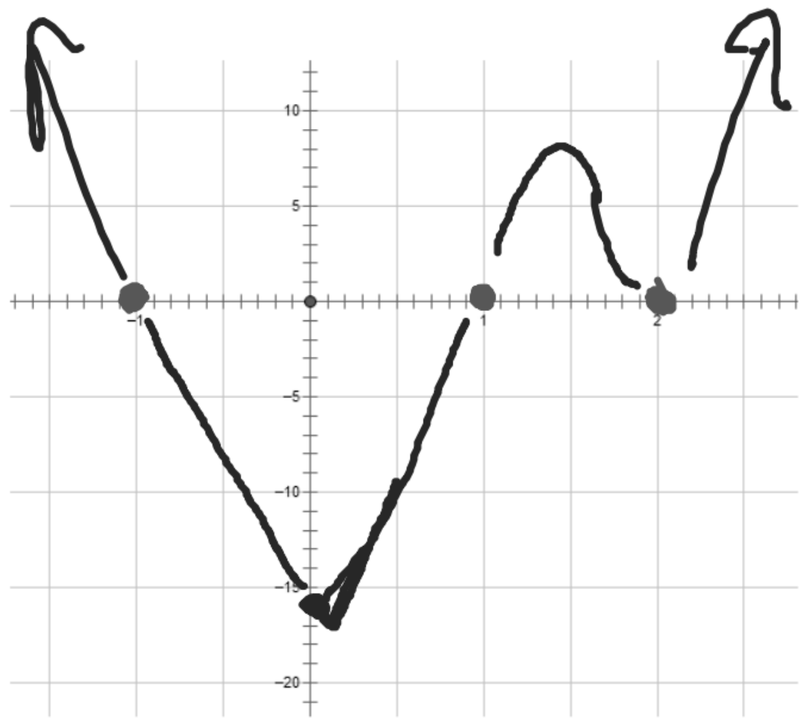
$\sqrt{5x+10}$

$$=\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{\sqrt{x^2}}\left(\sqrt{x^2+1}+x\right)}=\lim_{x\to\infty}\frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}+1)}$$

$$=\frac{\lim\limits_{x\to\infty}\frac{1}{x}}{\sqrt{\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}\frac{1}{x^2}}+\lim\limits_{x\to\infty}1)}=\frac{0}{\sqrt{1+0+1}}=0$$

$0 = (x-2)^4(x+1)^3(x-1)$

Sketch the graph of $y = (x-2)^4(x+1)^3(x-1)$ by finding its intercepts and its limits as x goes to pos and neg infinity.



- To find x-int: make y = 0
- To find y-int: make x = 0
- Regard signs when inputting -∞!

$\lim_{x \to \infty} f(x) = \infty \cdot \infty \cdot \infty = \infty$

$\lim_{x \to -\infty} f(x) = \infty \cdot (-\infty)(-\infty)$

$= \infty$

Homework 1.5 #'s 3, 4, 6, 10, 15 - 22 all, 29, 35, 41, 51

Hint:

#35 use squeeze theorem
(start with $-1 \leq \cos x \leq 1$)

Some practice problems. Just for funsies.

$$\lim_{x\to\infty}\frac{x^2+5}{(x-2)(3x+10)} \quad \boxed{\frac{1}{3}}$$

$$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)\cdot\frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}$$

$$\lim_{x\to\infty}\frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}$$

$$\lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{1}{\frac{1}{x}\sqrt{9x^2+x}+3}$$

$$= \lim_{x\to\infty}\frac{1}{\frac{1}{\sqrt{x^2}}\sqrt{9x^2+x}+3}$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{9\frac{x^2}{x^2}+\frac{x}{x^2}}+3}$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{9+\frac{1}{x}}+3}$$

$$= \frac{1}{\sqrt{9+0}+3} = \boxed{\frac{1}{6}}$$

YOU CAN TELL ITS $\frac{1}{6}$

$$\lim_{x\to\infty}\frac{x-5}{(2x^2-2)(3x+10)}$$

$$\lim_{x\to\infty}\frac{1}{6x^3} = \boxed{0}$$

$$\lim_{x\to\infty}\frac{x^3-4x+9}{(x-3)(x+11)} \quad \boxed{\infty}$$