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$$\lim_{h \rightarrow 0} \cos(a+h) = \cos a$$

$$\lim_{h \rightarrow 0} (\cos a \cdot \cos h - \sin a \cdot \sin h) = \cos a$$

$$\cos a \cdot (1 - \sin a \cdot 0) = \cos a$$

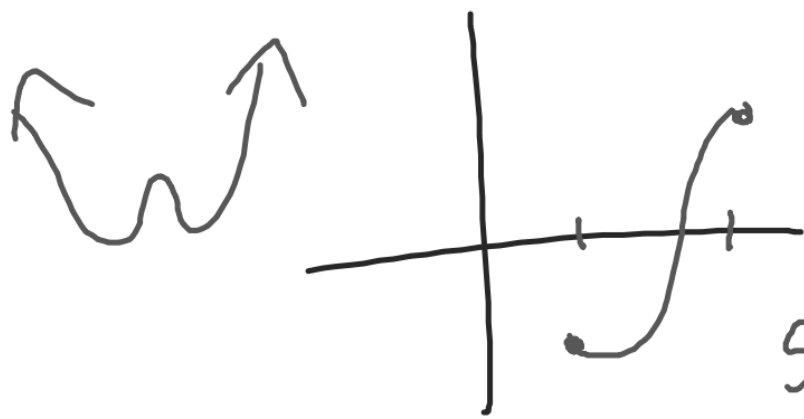
$$\cos a = \cos a$$



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$$x^4 + x - 3 = 0$$

$(1, 2)$



$$f(1) = 1^4 + 1 - 3 = -1$$

$$f(2) = 2^4 + 2 - 3 = 15$$

SINCE $f(1) < 0$ AND
 $f(2) > 0$ AND

$f(x)$ IS CONTINUOUS
THEN BY THE

INTERMEDIATE
VALUE THEOREM

$f(x)$ HAS A ROOT
ON $(1, 2)$.

$$x = x^3 + 1$$

$$0 = x^3 - x + 1$$

$$\text{IF } x = 0$$

$$f(0) = 0^3 - 0 + 1$$

$$f(0) = 1$$

SINCE $f(0) > 0$ AND

$f(-2) < 0$ AND

$f(x)$ IS CONTINUOUS,

BY THE IVT THERE IS

A VALUE x SUCH THAT

$$f(x) = 0.$$

$$\text{IF } x = -2$$

$$f(-2) = (-2)^3 - 2 + 1$$

$$= -8 + 2 + 1$$

$$= -5$$

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 7x^3 - 9}{x^2 - 15x + 12x^3}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{9+x^2} - 3x}{\sqrt{9+x^2} + 3x} \cdot \frac{\sqrt{9+x^2} + 3x}{\sqrt{9+x^2} + 3x}$$

$$\lim_{x \rightarrow \infty} \frac{9+x^2-9x^2}{\sqrt{9+x^2}+3x} = \lim_{x \rightarrow \infty} \frac{-8x^2+9}{\sqrt{9+x^2}+3x}$$