Do Now - Sept. 8th, 2017

Lemme check the Crafty Limit sheet, then try this:

Prove that...

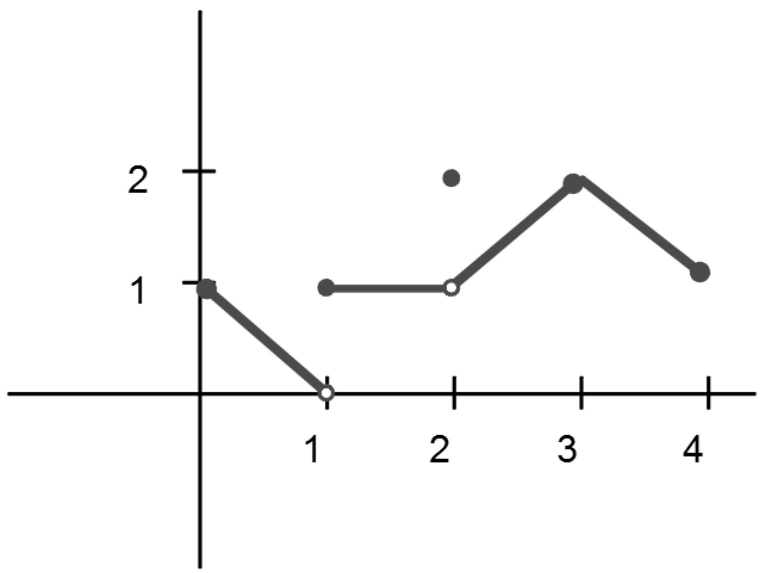
$$\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$$

2.5 Continuity

Most of the techniques of calculus require that functions be continuous. A function is continuous if you can draw it in one motion without picking up your pencil.

A function is continuous at a point if the **limit** is the same as the **value** of the function.

$$\lim_{x \to a} f(x) = f(a)$$



This function has <u>discontinuities</u> at x=1 and x=2.

It is continuous at x=0 and x=4, because the one-sided limits match the value of the function.

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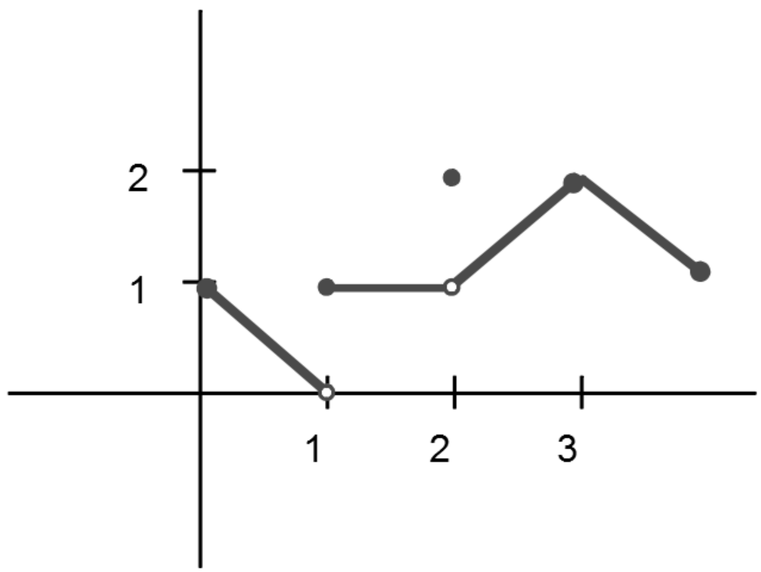


At x = 1, this function is continuous from the right

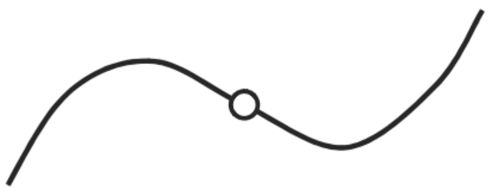
At x = 2, it is continuous from neither side

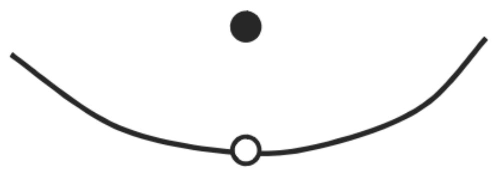
Knowing this, finish the equation:

If a function is continuous from the left, then

$$\lim_{x \to a^-} f(x) = $$ [redacted]

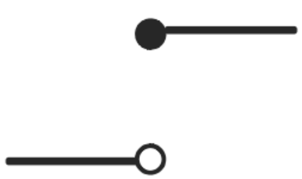
Removable Discontinuities:



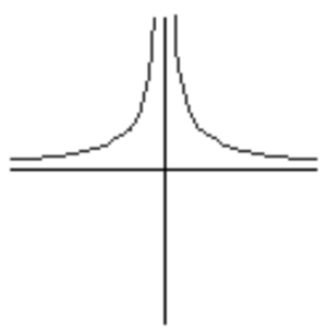


(You can fill the hole.)

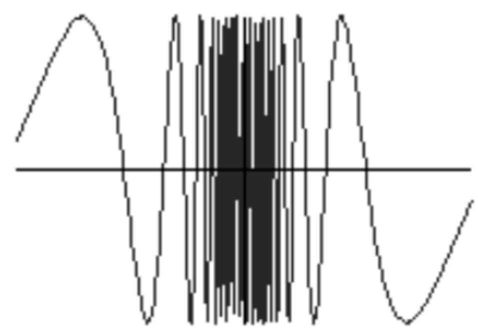
Essential Discontinuities:



jump



infinite



oscillating

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Removing a discontinuity:

$$f(x)=\frac{x^3-1}{x^2-1}$$

has a discontinuity at $x=1$.

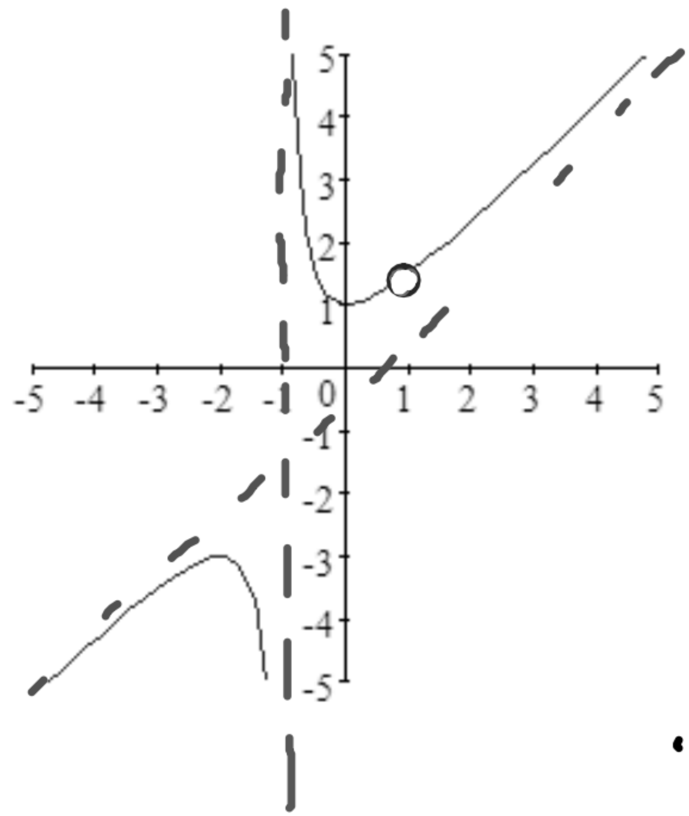
Write an extended function that is continuous at $x=1$.

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}=\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{(x+1)\cancel{(x-1)}}=\frac{1+1+1}{2}=\frac{3}{2}$$

$$f(x)=\begin{cases}\dfrac{x^3-1}{x^2-1}, & x\neq 1\\[2mm] \dfrac{3}{2}, & x=1\end{cases}$$

Note: There is another discontinuity at $x=-1$ that can not be removed.

Removing a discontinuity



$$f(x)=\begin{cases}\dfrac{x^3-1}{x^2-1}, & x\neq 1\\[2ex] \dfrac{3}{2}, & x=1\end{cases}$$

Note: There is another discontinuity at $x=-1$ that can not be removed.

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Continuous functions can be added, subtracted, multiplied, divided and multiplied by a constant, and the new function remains continuous.

Also: Composites of continuous functions are continuous.

examples: $y = \sin\left(x^2\right)$ $y = \left|\cos x\right|$

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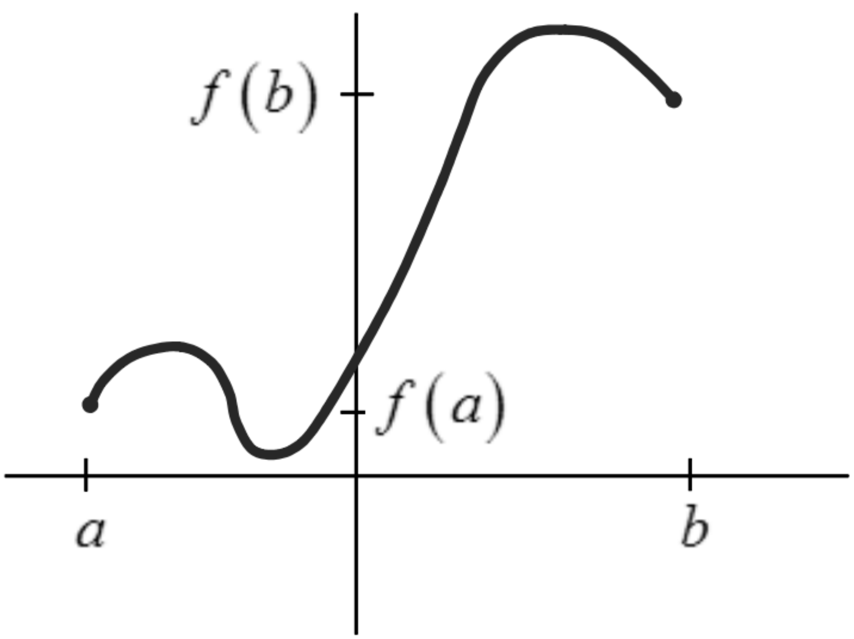
Intermediate Value Theorem

If a function is continuous between a and b, then it takes on every value between $f(a)$ and $f(b)$.



Because the function is continuous, it must take on every y value between $f(a)$ and $f(b)$.

→

Example 5: Is any real number exactly one less than its cube?

(Note that this doesn't ask what the number is, only if it exists.)

$$x = x^3 - 1$$

$$0 = x^3 - x - 1$$

$$f(x) = x^3 - x - 1$$

$$f(1) = -1 \qquad f(2) = 5$$

Since f is a continuous function, by the intermediate value theorem it must take on every value between -1 and 5.

Therefore there must be at least one solution between 1 and 2.

Homework:

Be sure to read through the book and familiarize yourself with the various theorems and definitions related to continuity.

Hw 2.5
#'s 3, 17, 21, 33, 39, 41, 47, 57, 61

Hint for # 57:
Well, look at # 56. But for # 57, start with the definition of continuity:

$$\lim_{h \to 0} \cos(a+h) = \cos(a)$$

Then use the trigonometric "addition" formula from Precalc. Don't remember it? Look at the reference page at the front of your book.

Finally, apply limit laws to the left side of the equation to make it look like the right side.