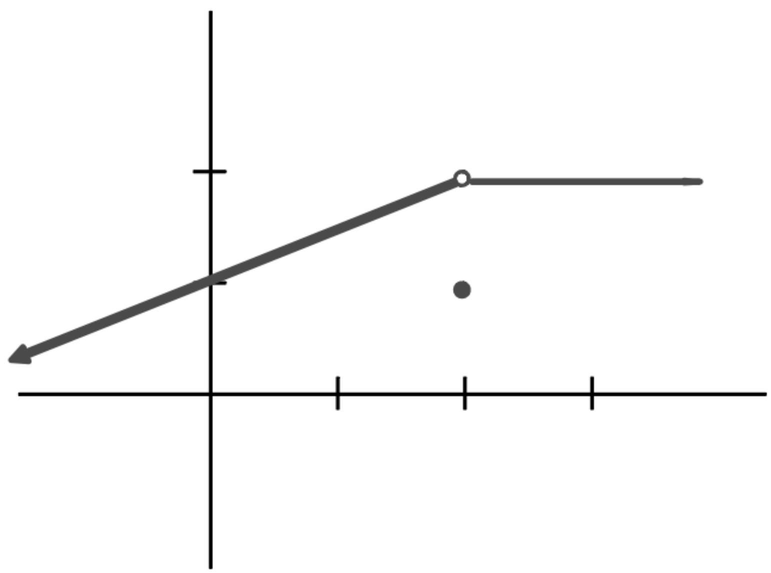
BC Calc 1.2 - The Limit of a Function

$$\lim_{x \to a} f(x) = L$$

The limit of f(x), as x approaches *a*, equals L

f(x) gets as close as we want to L, as x gets close to a

The <u>limit</u> of a function refers to the value that the function <u>approaches</u>, <u>not</u> the actual value (if any).

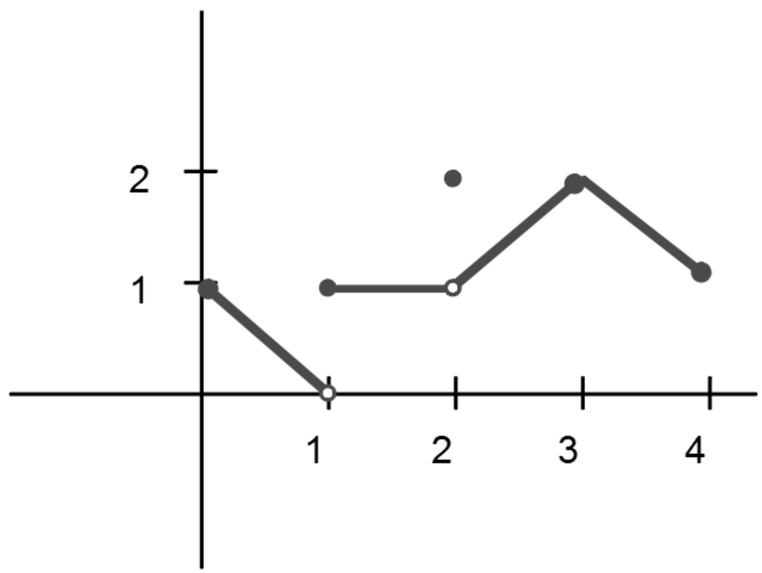


$$\lim_{x \to 2} f(x) = 2$$

<u>not 1</u>

For a limit to exist, the function must approach the same value from both sides.

One-sided limits approach from either the left or right side only.



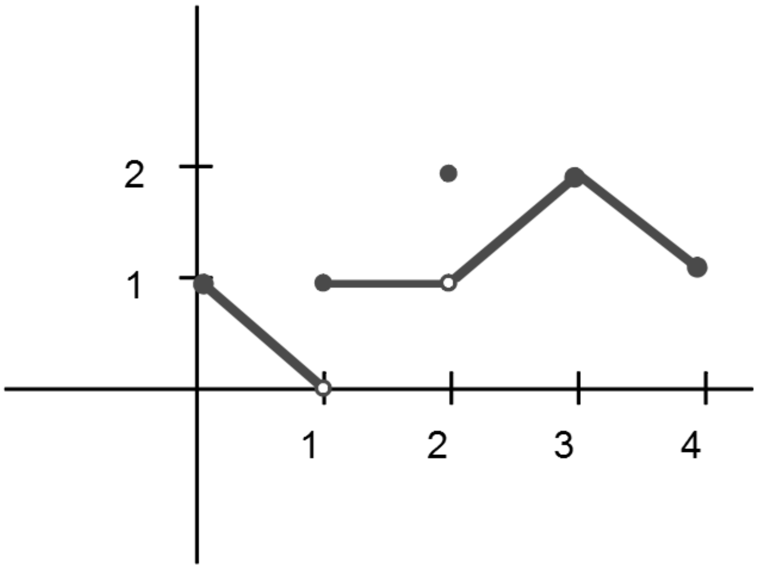
$\lim_{x\to 1} f(x)$ does not exist because the left and right hand limits do not match!

At x=1: $\lim_{x\to 1^-} f(x) = 0$ —— left hand limit

$\lim_{x\to 1^+} f(x) = 1$ —— right hand limit

$f(1) = 1$ —— value of the function

→



$$\lim_{x\to 2} f(x) = 1$$

because the left and right hand limits match.

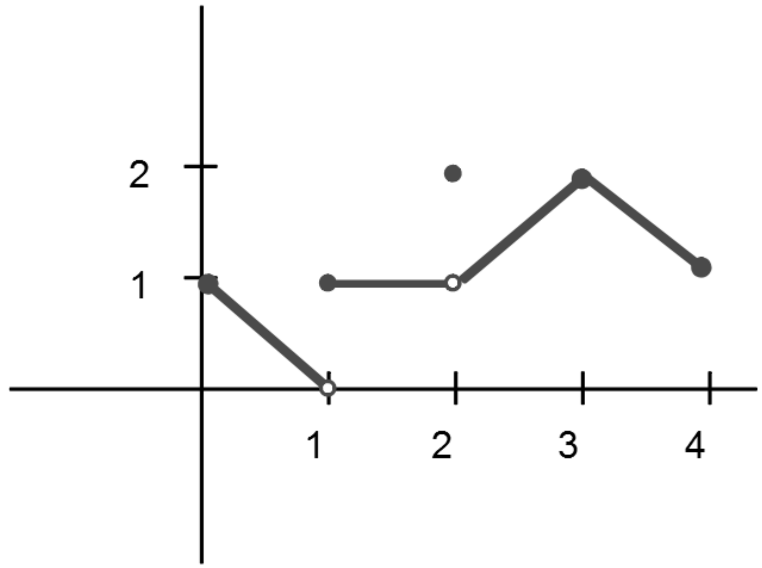
At x=2:

$$\lim_{x\to 2^-} f(x) = 1$$ — left hand limit

$$\lim_{x\to 2^+} f(x) = 1$$ — right hand limit

$$f(2) = 2$$ — value of the function

→



$$\lim_{x\to 3} f(x) = 2$$

because the left and right hand limits match.

At x=3: $\lim_{x\to 3^-} f(x) = 2$ — left hand limit

$\lim_{x\to 3^+} f(x) = 2$ — right hand limit

$f(3) = 2$ — value of the function

→

Try this:

$$\lim_{x \to 1} \frac{x-1}{x^2-1}$$

This function isn't defined at x = 1, but that's OK, since a limit has to do with the values around *a* but not equal to *a*.

So, let's try to put some values in around 1.

$x < 1$	f(x)
0.5	0.666667
0.9	0.526316
0.99	0.502513
0.999	0.500250
0.9999	0.500025

$x > 1$	f(x)
1.5	0.400000
1.1	0.476190
1.01	0.497512
1.001	0.499750
1.0001	0.499975

So according to this, a good guess would be $\lim_{x \to 1} \frac{x-1}{x^2-1} = 0.5$

Limits do not exist if the same value is not approached from both sides of *a*. Another case where the limit does not exist is when a function oscillates where the function is being evaluated.

$$\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$$

f(1) = sin π = 0

f(1/2) = sin 2π = 0

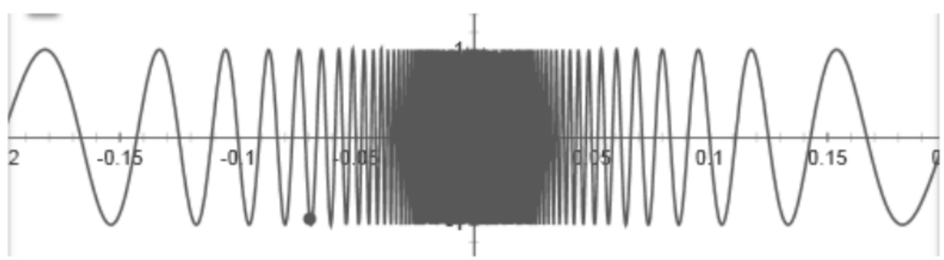
f(1/3) = sin 3π = 0

f(1/4) = sin 4π = 0

f(0.1) = sin 10π = 0

f(0.01) = sin 100π = 0

Sure looks like 0, doesn't it? But try looking at the graph:



The limit does not exist!

Finally, a limit does not exist if as *x* approaches *a* f(x) approaches ∞.

$$\lim_{x \to 0} \frac{1}{x^2}$$

Trying large numbers (or in this case simply looking at the graph) we can see that the limit is ∞.

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$

Not that ∞ is actually a number, just that f(x) can get as large as we want it to be as *x* approaches 0.

"f(x) increases without bound as *x* approaches *a* "

or

"the limit of f(x) as *x* approaches *a* is infinity"