Part 1: Using Vectors to Approximate the Neutral Current in a Three Phase Power System

for three phase 60 hertz systems with unity power factor and resistive loads

by Gerald Newton
November 3, 1999

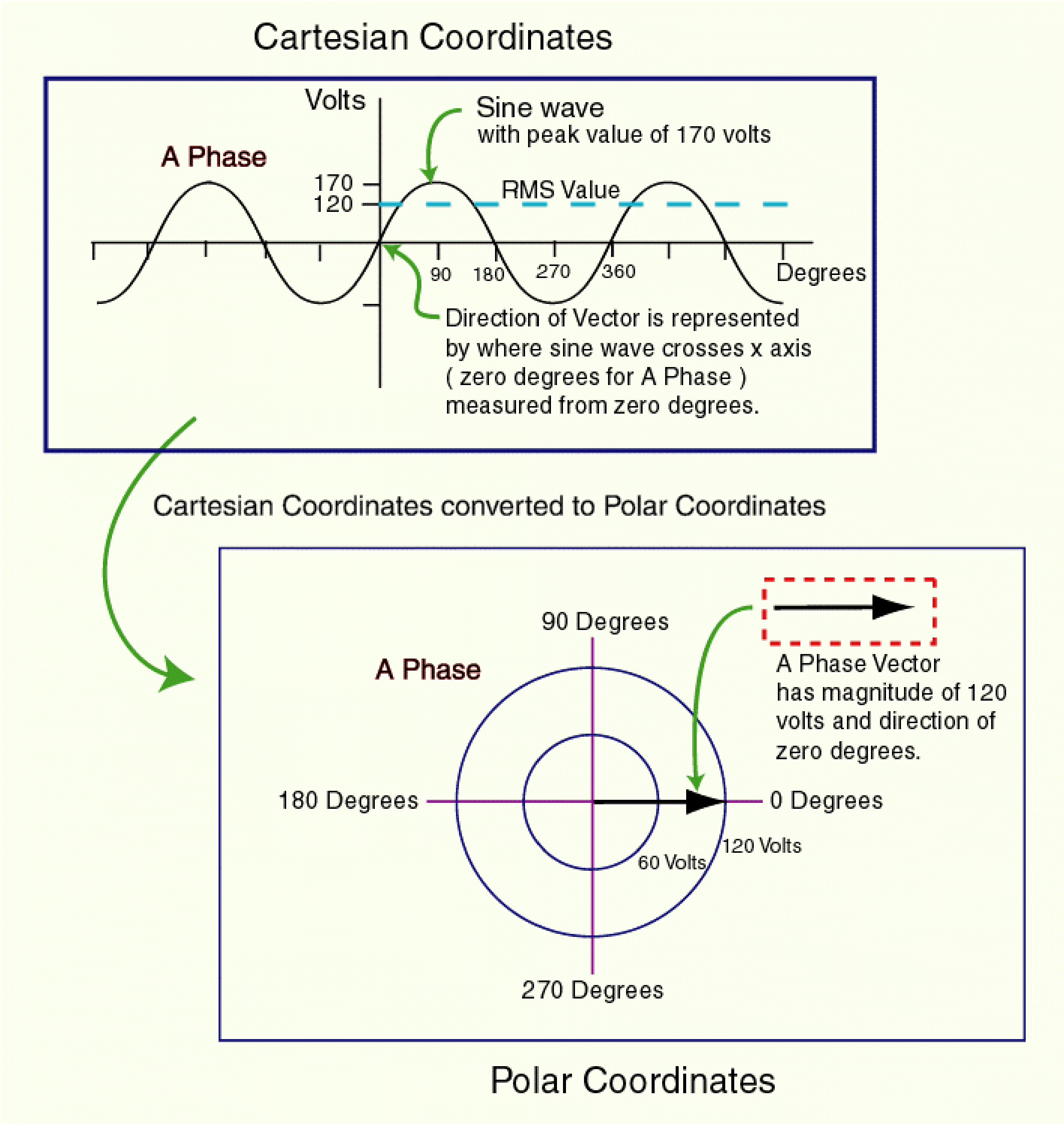
The diagrams below show just how simple three phase power can be. American electricians are fortunate that power frequencies are established at 60 hertz and take the form of the common sine wave. Engineers, on the other hand, are quite unfortunate since they must learn electrical theory for any frequency and wave form, and for power factors other than unity. But this article is not for engineers, it is for the American electrician so calculus is not going to be included in this discussion. However, electricians are expected to know geometry, trigonometry, algebra and arithmetic. The diagrams below are an attempt to provide a simple understanding of three phase power systems such as seen with 208/120 volt and 480/277 volt three phase, 4 wire systems. We already know by common electrical knowledge that the neutral current in a balanced three phase system is 0 amperes for resistive loads such as incandescent lighting and heaters where the power factor is approximately 1.0. We also know that in a balanced three phase system the neutral current is approximately equal to the line current for electric discharge lighting loads such as high pressure sodium lights or fluorescent lights because of third harmonics.

For systems where the line currents are not equal there is a simple method for finding the neutral current. We can convert A phase, B phase and C phase into polar coordinates where the amperes are represented by vectors then add the three vectors to find the approximate neutral current and phase angle. This is called geometric vector addition.

Mathematically, we can precisely add the three vectors for the line currents by converting vectors into their horizontal and vertical components by using trigonometry. We can add the horizontal components, then add the vertical components, then convert this to polar coordinates to find the sum of the vectors. This is called algebraic vector addition.

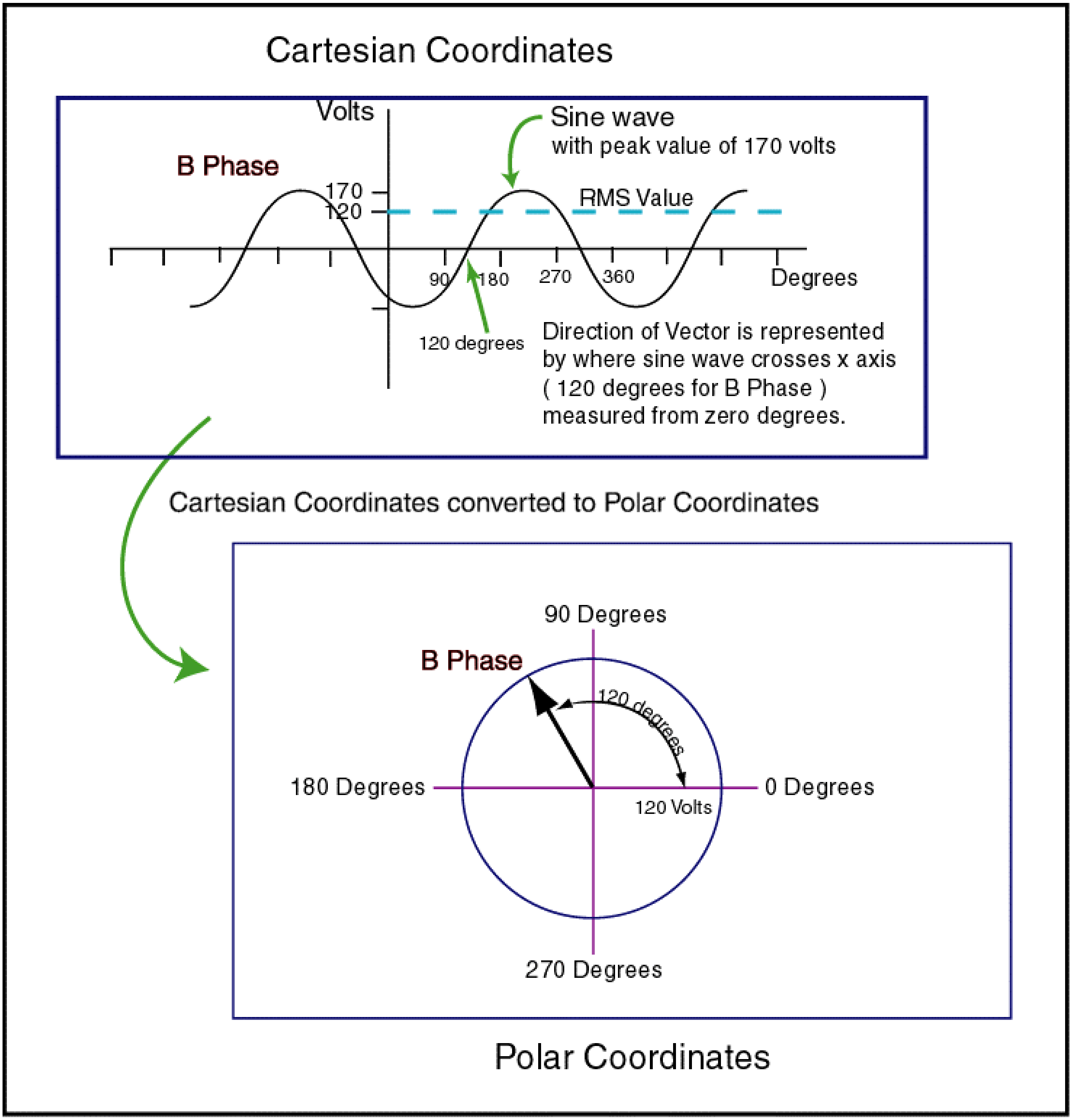
We will do the geometric addition using the root mean square values of the sine wave. The second part of this discussion will cover algebraic addition.

First let us convert cartesian coordinates or the familiar X Y type graph to a Polar graph as demonstrated below.



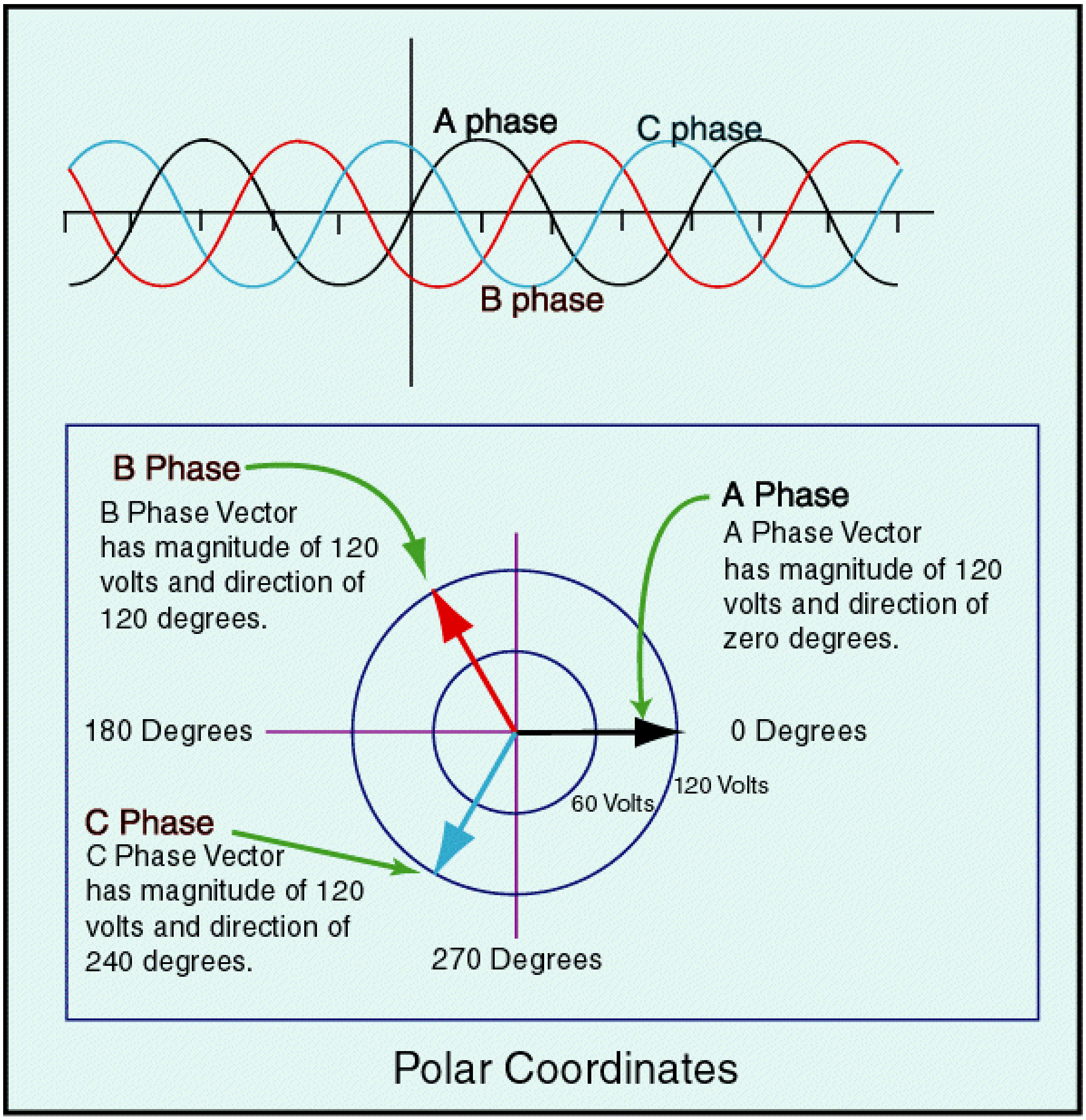
Cartesian Coordinates
Volts
Sine wave
with peak value of 170 volts
A Phase
170
120
RMS Value
90
180
270
360
Degrees
Direction of Vector is represented by where sine wave crosses x axis (zero degrees for A Phase) measured from zero degrees.
Cartesian Coordinates converted to Polar Coordinates
90 Degrees
A Phase
A Phase Vector has magnitude of 120 volts and direction of zero degrees.
180 Degrees
0 Degrees
120 Volts
60 Volts
270 Degrees
Polar Coordinates

So what about B Phase? Well here it is!



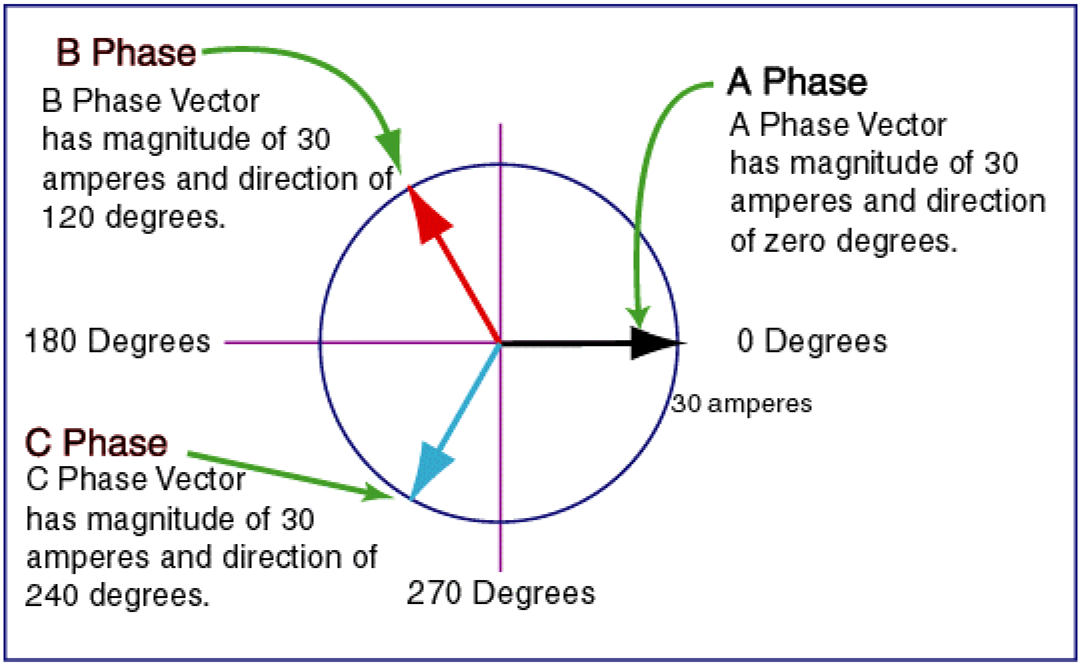
Cartesian Coordinates
Volts
Sine wave
with peak value of 170 volts
B Phase
170
120
RMS Value
90
180
270
360
Degrees
120 degrees
Direction of Vector is represented by where sine wave crosses x axis (120 degrees for B Phase) measured from zero degrees.
Cartesian Coordinates converted to Polar Coordinates
90 Degrees
B Phase
120 degrees
180 Degrees
0 Degrees
120 Volts
270 Degrees
Polar Coordinates

Combining A phase, B phase, and C Phase.....

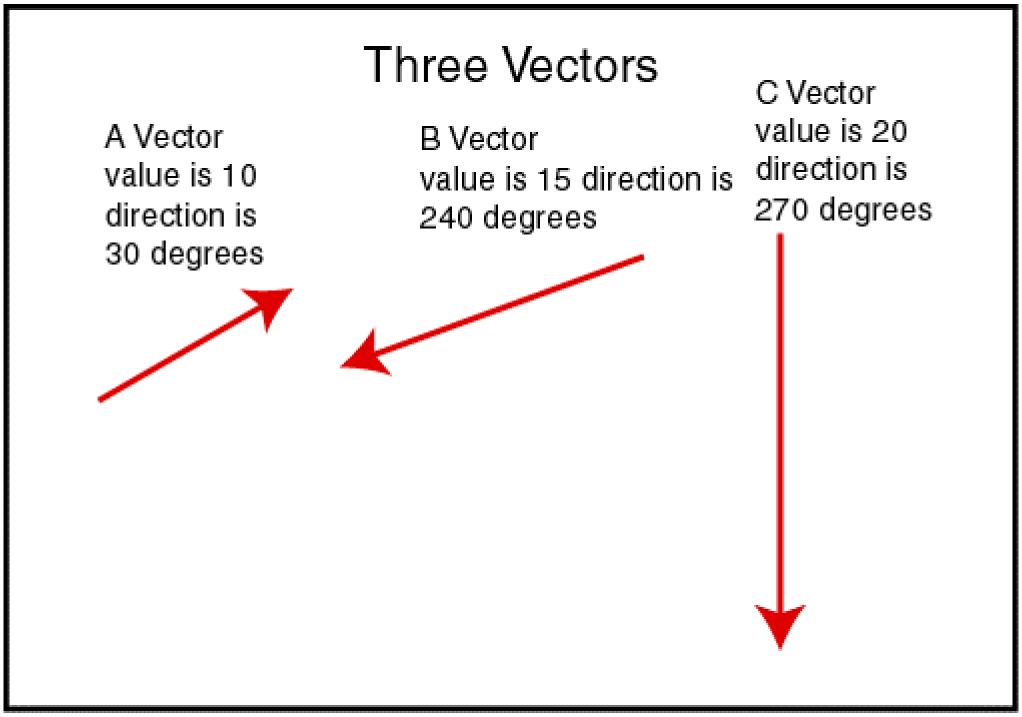


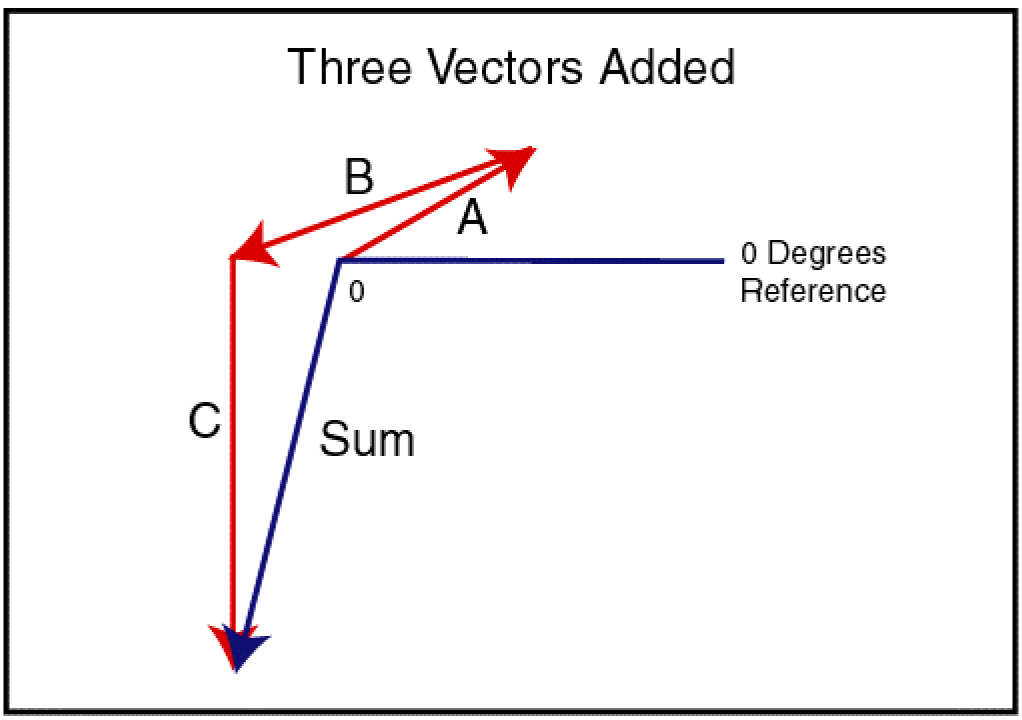
That looks easy enough and is. We have all learned a little about vectors, I am sure. A Vector is a line that represents magnitude (its length) and has direction. The direction is referenced to zero degrees and that is usually a horizontal line that parallels this sentence.

For example below is the phase vectors representing a line current of 30 amperes for each phase.



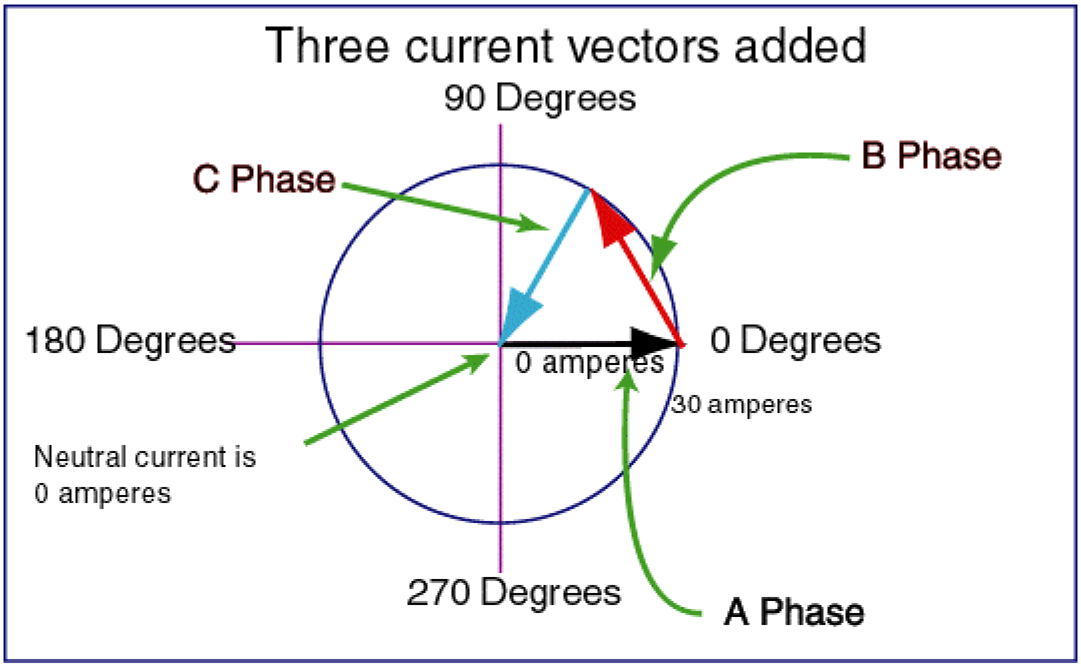
Our next step is to add the vectors. The neutral current is the sum of the A phase, B phase, and C phase vectors. So how do we add vectors? Actually, there is a very simple, easy, and fast method that gives a good approximation. Simply place each vector on the end of the preceding vector, then draw a line from the center point where the magnitude value is zero to the end of the final vector. It is done like this:



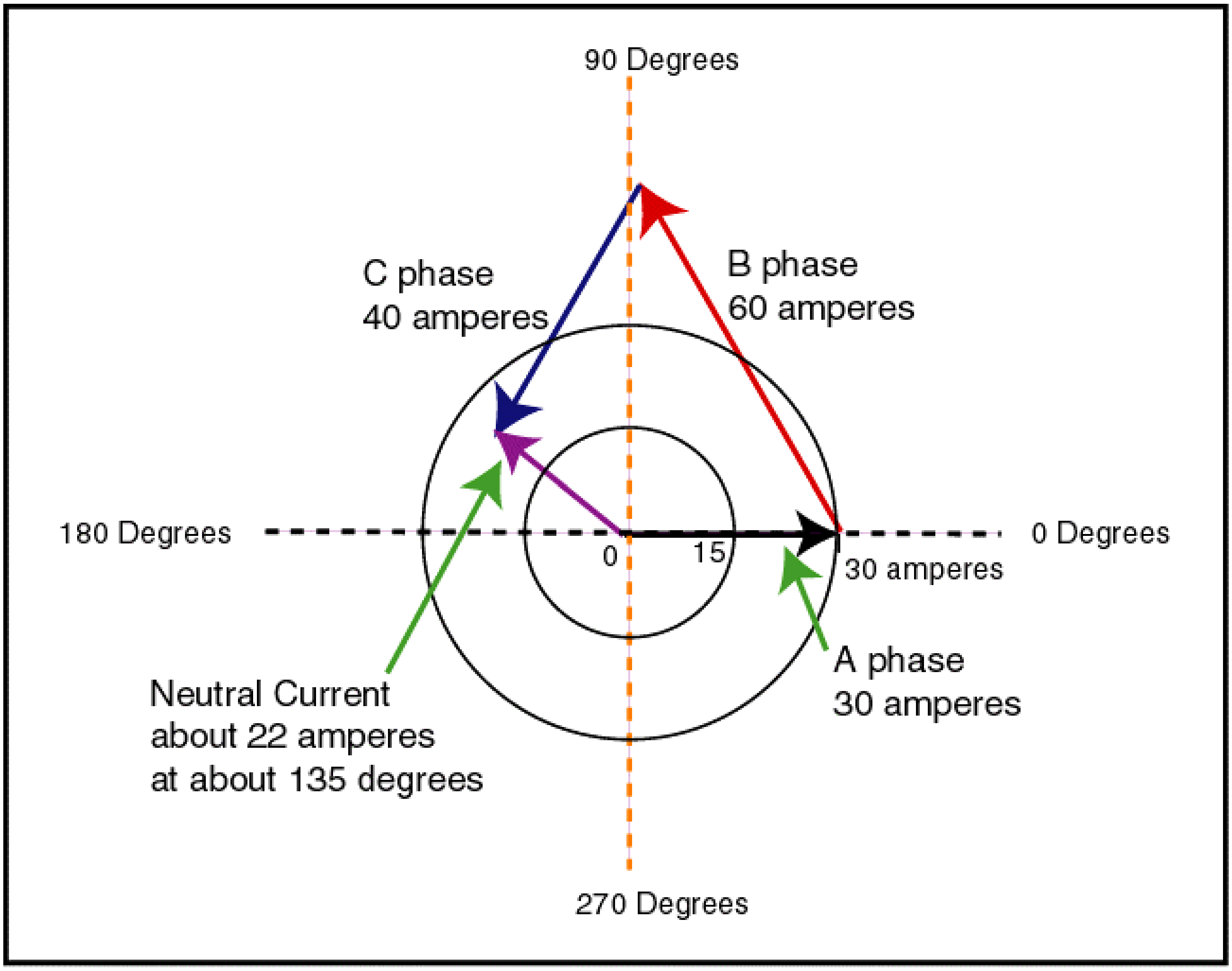


The approximate value of the Sum is 20 plus something and the direction is about 260 degrees. We could get much better results using polar graph paper and a ruler to scale the values.

We can do the same thing with the three phase vectors that represent current as shown below.
In this example the vectors represent equal line currents of 30 amperes. The neutral current should be zero and sure enough, the sum of the phase currents add up to a vector of zero magnitude. The direction is irrelevant. Remember, the neutral current is the sum of the three vectors that represent the phase line currents.



Now let us try this approximation method to add three phase line currents that are not equal.



Click the mouse button to place several vectors on the screen. Then click on action to sum them up. The vector is the sum is the white vector from the origin to the head of the last vector.

Use the mouse button to lay down up to ten vectors. Click on the Action button to sum the up.

To go to Part 2 using algebra to add vectors click here.

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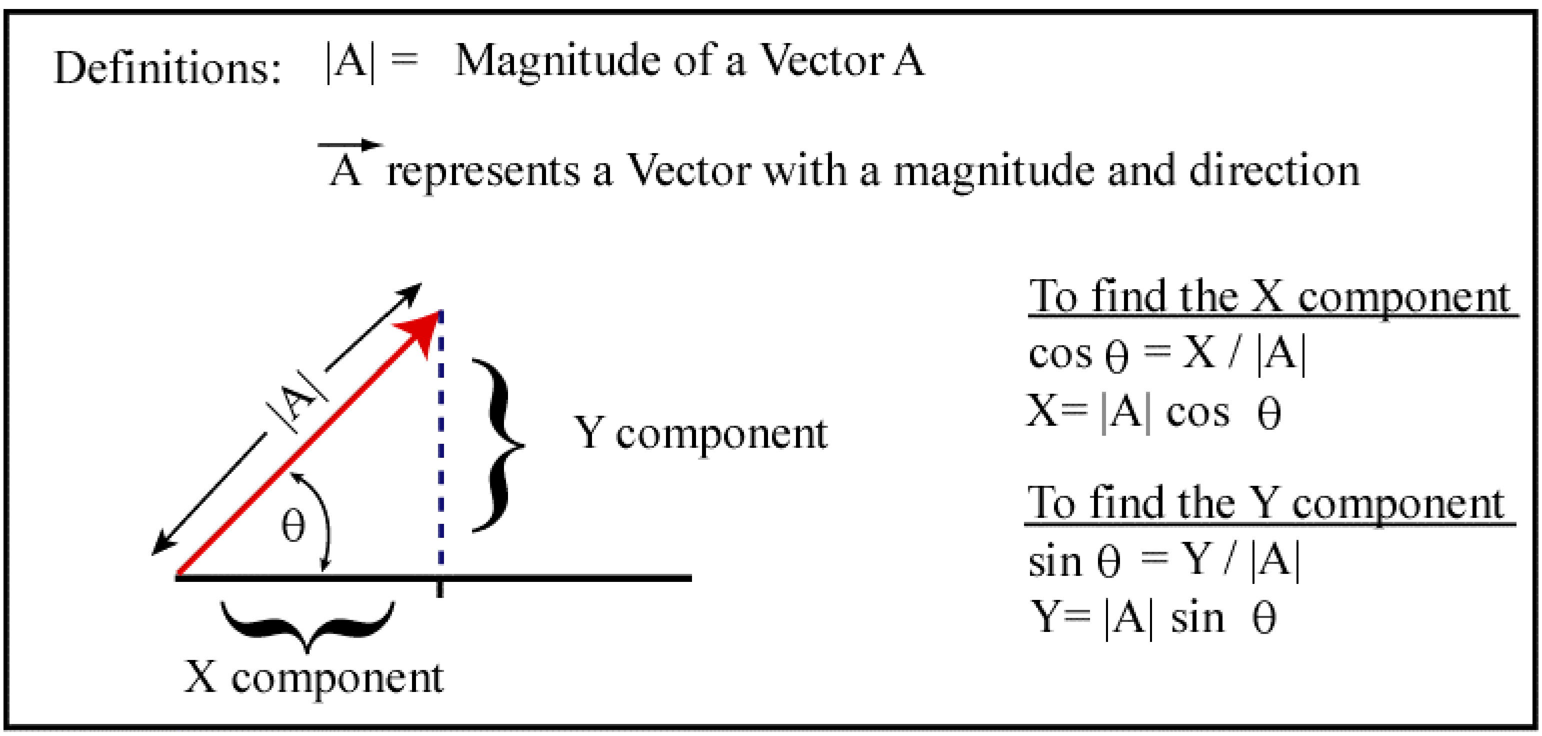
Part 2: Using Vectors to Approximate the Neutral Current in a Three Phase Power System

for three phase 60 hertz systems with unity power factor and resistive loads

by Gerald Newton
November 5, 1999

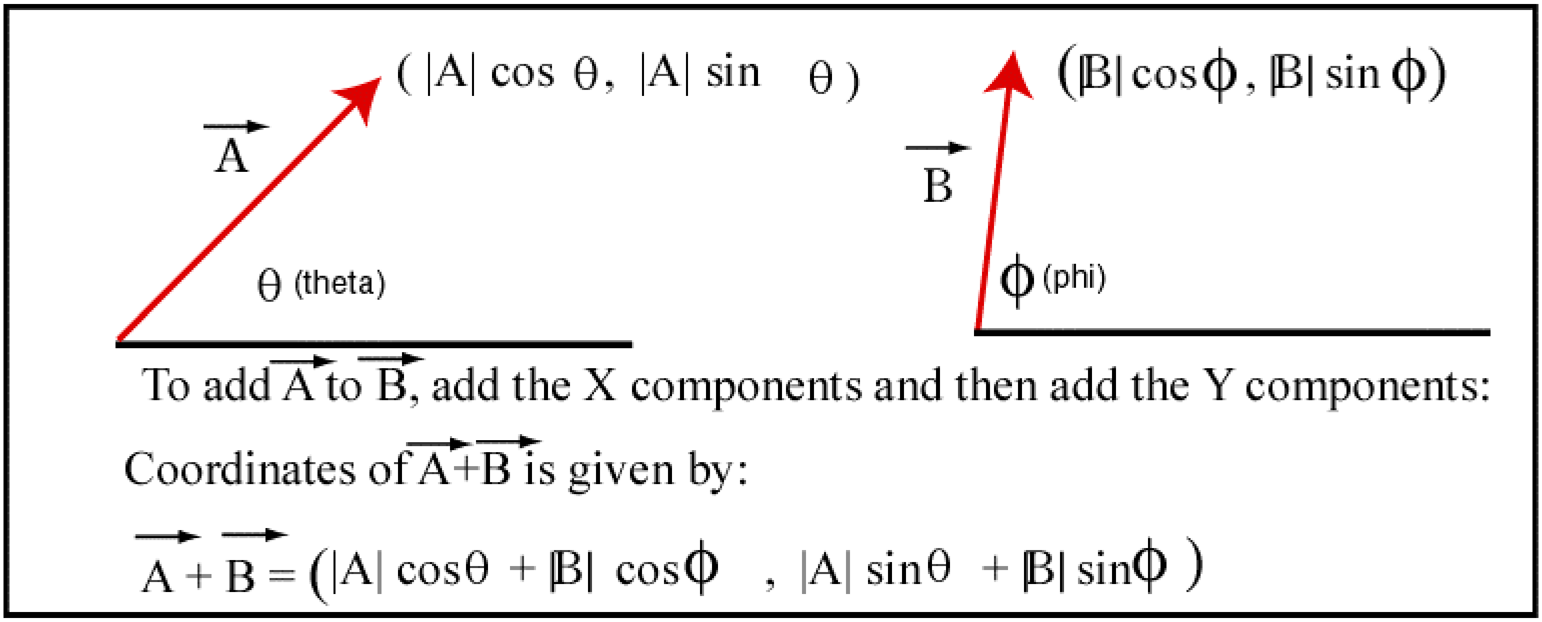
The geometric addition of vectors gives a fast approximation of neutral current in a three phase system. A more precise calculation can be performed using algebra. This can be a long mathematical problem, but by restricting our calculation to A phase at 0 degrees, B phase at 120 degrees, and C phase at 240 degrees the calculation is simplified.

The figure below demonstrates how to break a vector down into its horizontal (X) and vertical (Y) components. The magnitude of the vector is its length in

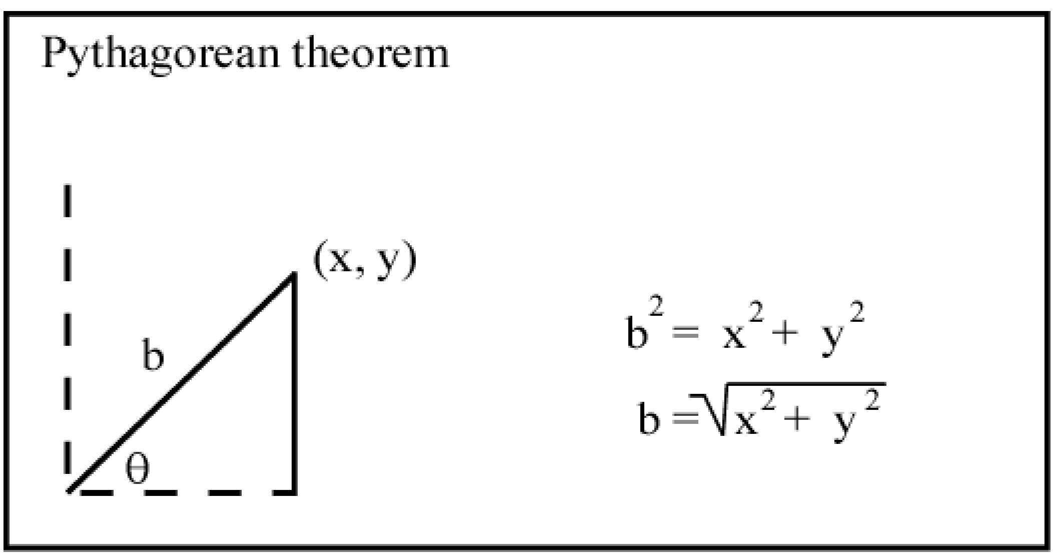


whatever units we chose to represent. In our case that would be amperes.

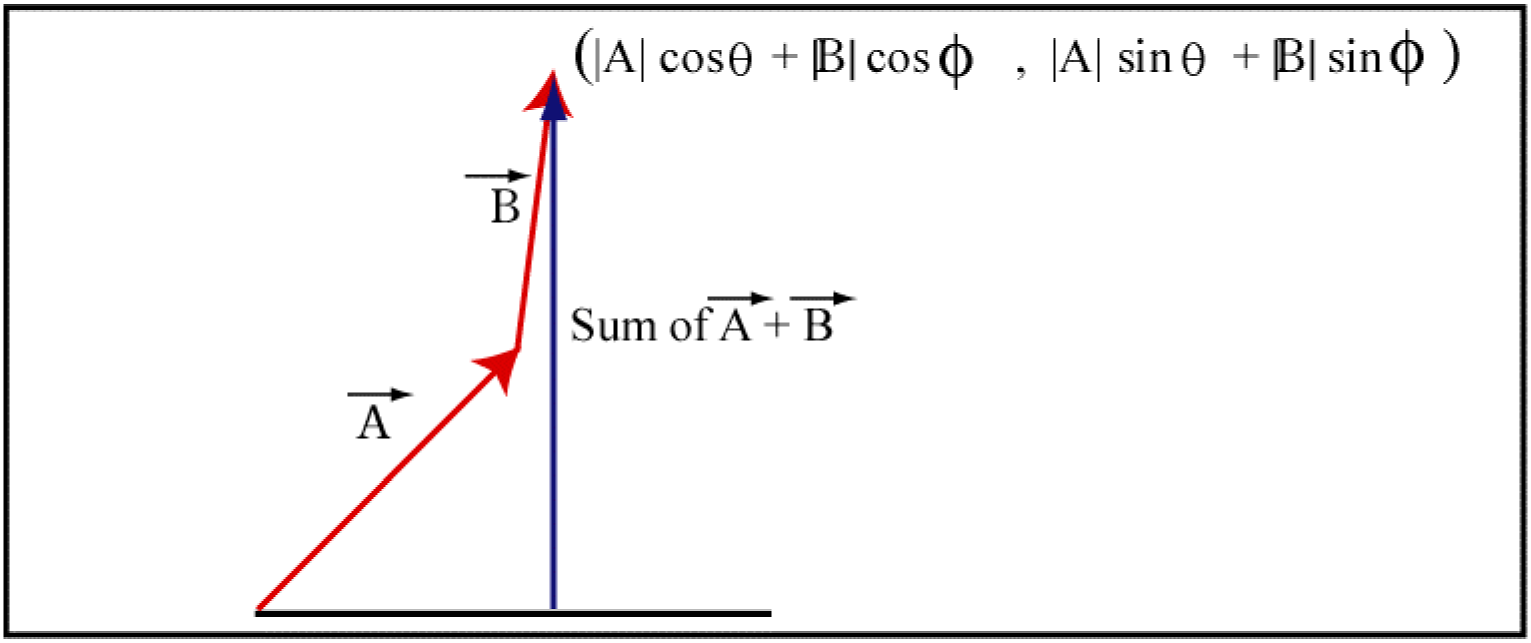
The following figures demonstrates the algebraic addition of two vectors.

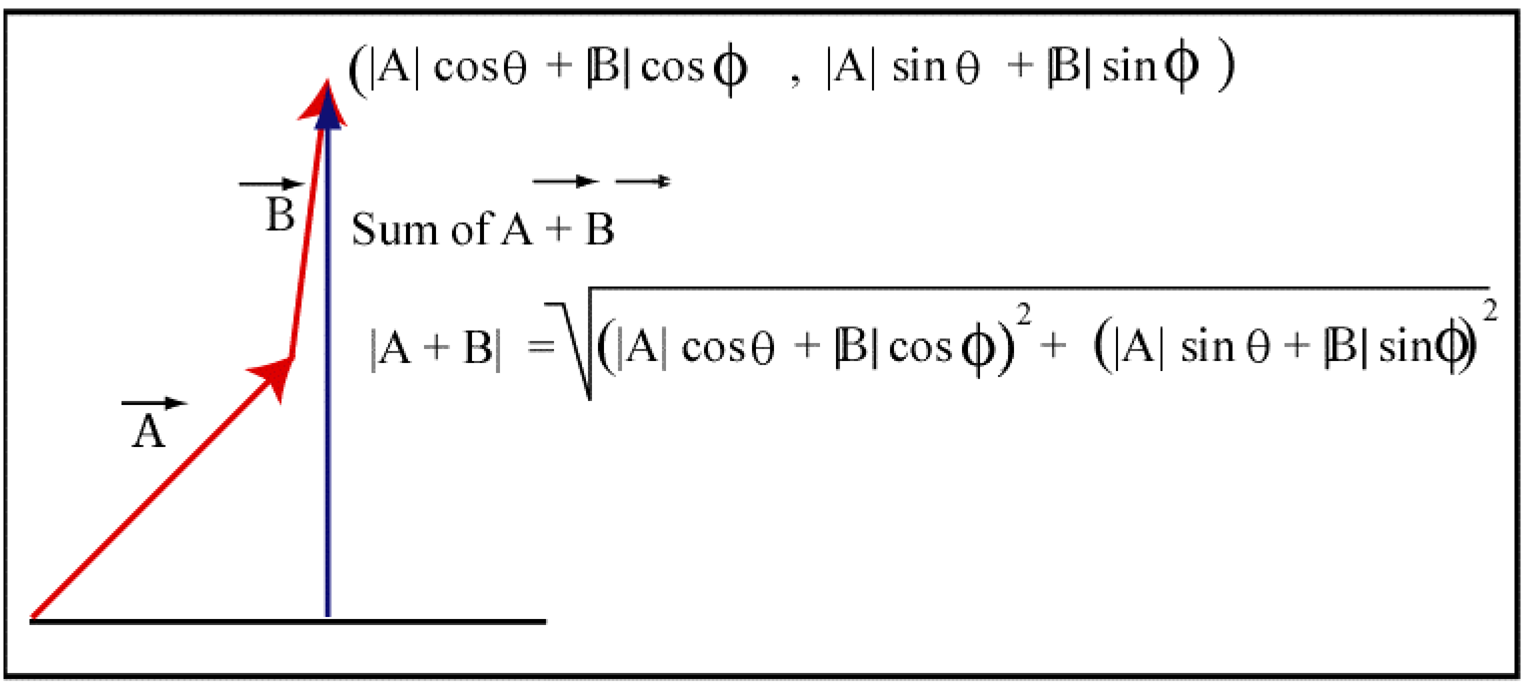


The Pythagorean is used to find the algebraic value of a vector once its X and Y components are known.



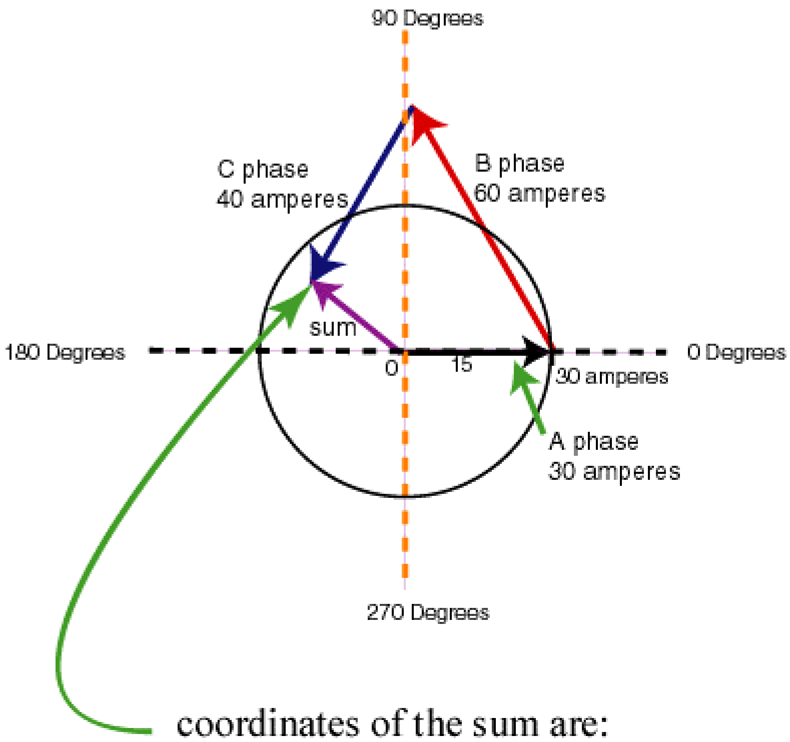
The following demonstrates the use of the Pythagorean theorem to find the magnitude of the sum of two vectors.





For three vectors the magnitude calculation is similar to that for two vectors. A third component is added for the third vector. The process of multiplying the polynomials can be reduced by using the known angles for the A phase, B phase, and C phase vectors. The sines and cosines are

known for the three angles, 0 degrees, 120 degrees, and 240 degrees.



Remember our guess using geometric addition was:
Neutral Current
about 22 amperes
at about 135 degrees

coordinates of the sum are:

$$|A| \cos\theta + |B| \cos\phi + |C| \cos\gamma \,)\ ,\ (\,|A| \sin\theta + |B| \sin\phi + |C| \sin\gamma\,)$$

Sum of $\vec{A} + \vec{B} + \vec{C}$ by Pythagorean theorem is given by:

$$|A + B + C| = \sqrt{(|A| \cos\theta + |B| \cos\phi + |C| \cos\gamma)^2 + (|A| \sin\theta + |B| \sin\phi + |C| \sin\gamma)^2}$$

But: cos 0 degrees is 1, cos 120 degrees is -.5, and cos 240 degrees is -.5
sin 0 degrees is 0, sin 120 degrees is .866, and sin 240 degrees is -.866

So

$$|A| \cos\theta + |B| \cos\phi + |C| \cos\gamma\,)^2 + (\,|A| \sin\theta + |B| \sin\phi + |C| \sin\gamma\,)^2$$ becomes

$$|A| \times 1 + |B| \times -.5 + |C| \times -.5)^2 + (\,|A| \times 0 + |B| \times .866 + |C| \times -.866\,)^2$$

and

$$|A + B + C| = \sqrt{|A| \times 1 + |B| \times -.5 + |C| \times -.5)^2 + (\,|A| \times 0 + |B| \times .866 + |C| \times -.866\,)^2}$$

substituting 30 amperes for |A|, 60 ampere for |B|, and 40 amperes for |C|

$$|A + B + C| = \sqrt{(30 \times 1 + 60 \times -.5 + 40 \times -.5)^2 + (\,30 \times 0 + 60 \times .866 + 40 \times -.866\,)^2}$$

$$|A + B + C| = \sqrt{-20^2 + 17.3^2}$$

$$|A + B + C| = \sqrt{700}$$

|A + B + C| = 26.5 amperes at **coordinates** (-20, 17.3)

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Use the mouse button to lay down up to ten vectors. Click on the Action button to calculate the sum.

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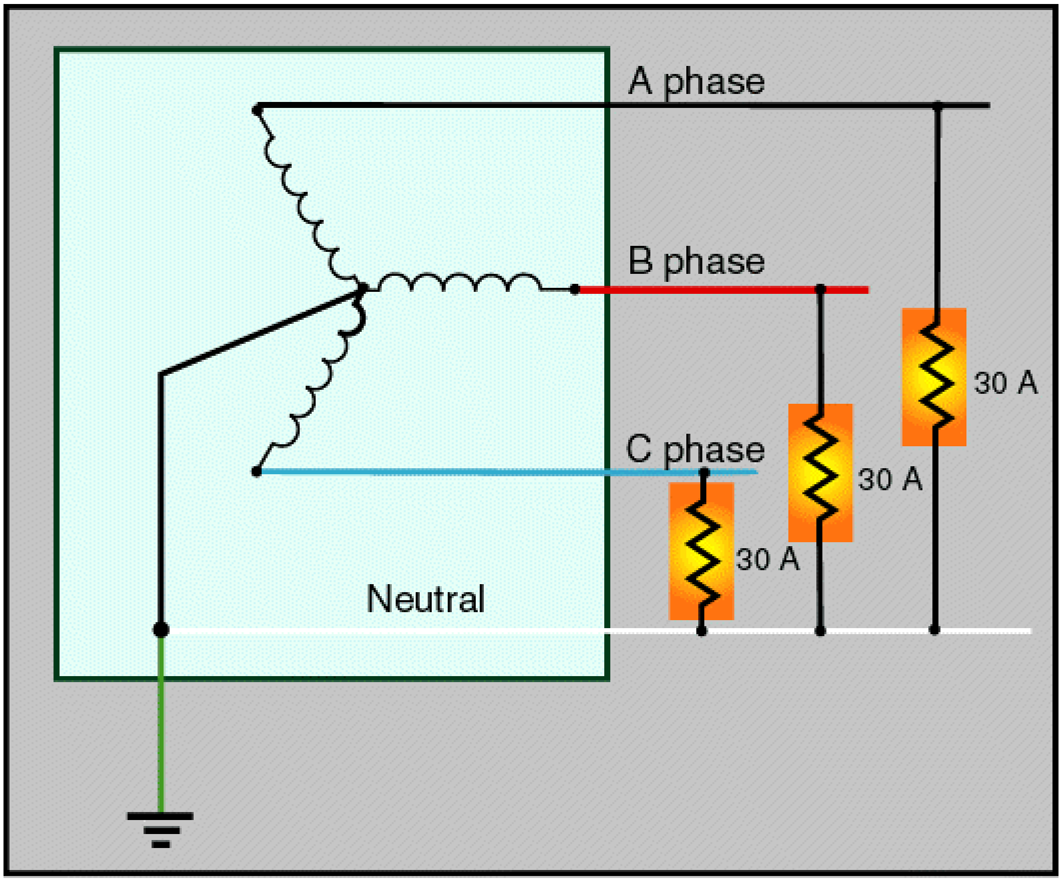
Part 3: Using Vectors to Approximate the Neutral Current in a Three Phase Power System

for three phase 60 hertz systems with unity power factor and resistive loads

by Gerald Newton
November 9, 1999

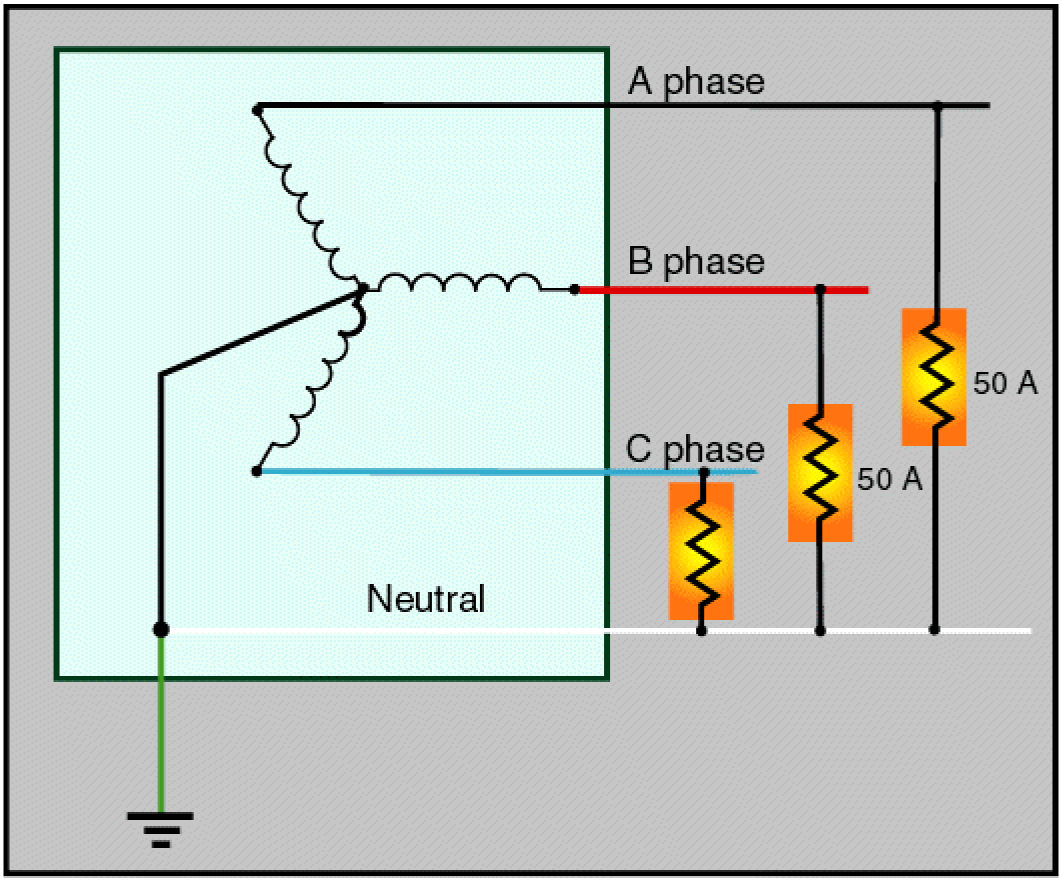
Using the tools from Part 1and Part 2 solve the following problems.

1. In the diagram below if the loads are electric heaters how much current would the neutral carry?



In a balanced three phase system supplying resistive loads the power factor is approximately 1 and the current in the neutral is 0 amperes.

2. In the diagram below if the C phase line current is 30 amperes what would the current be in the neutral if the loads are electric heaters?



$$|A+B+C| = \sqrt{\left(|A|\cos\theta + |B|\cos\phi + |C|\cos\gamma\right)^2 + \left(|A|\sin\theta + |B|\sin\phi + |C|\sin\gamma\right)^2}$$

Using the formula from the previous discussion.

|A| = 50 amperes, |B| = 50 amperes, |C| = 30 amperes and

cos 0 degrees is 1, cos 120 degrees is -.5, and cos 240 degrees is -.5

sin 0 degrees is 0, sin 120 degrees is .866, and sin 240 degrees is -.866

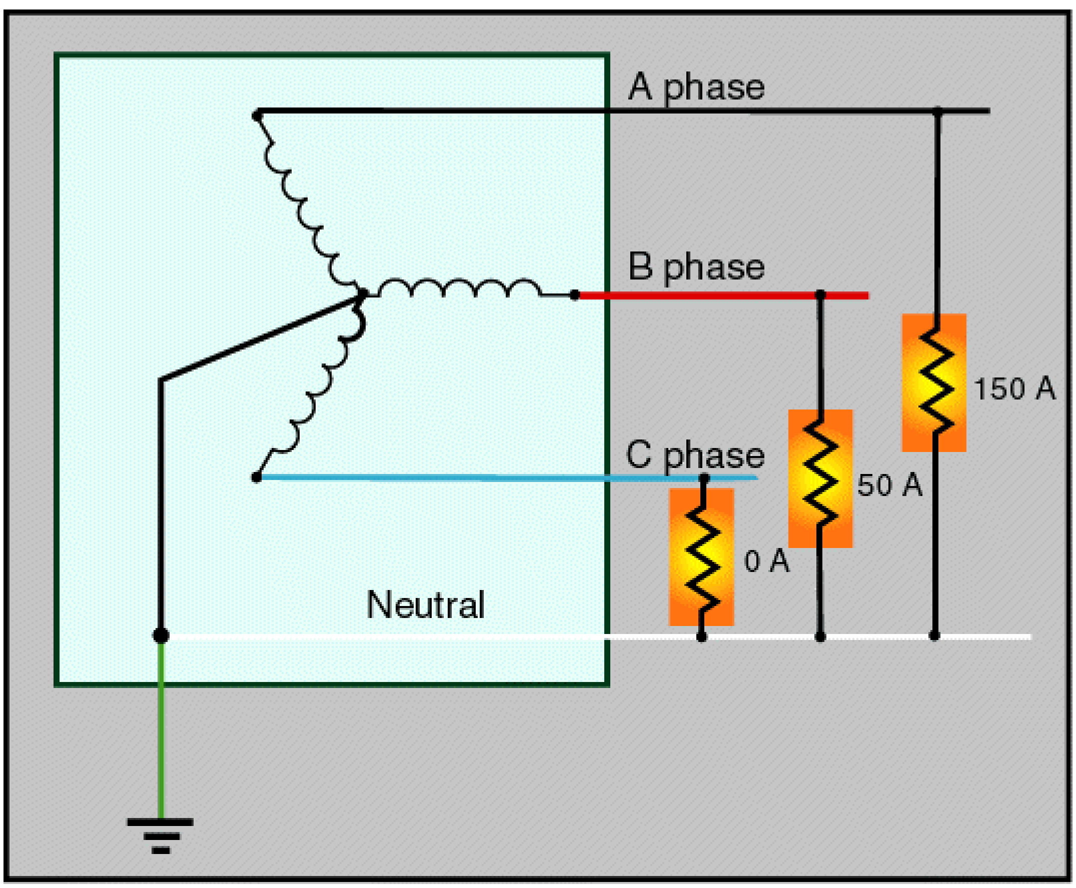
$$|A+B+C| = \sqrt{(50 \times 1 + 50 \times -.5 + 30 \times -.5)^2 + (50 \times 0 + 50 \times .866 + 30 \times -.866)^2}$$

$$|A+B+C| = \sqrt{10^2 + 17.3^2}$$

$$|A+B+C| = \sqrt{399}$$

$|A+B+C| =$ 20 amperes at coordinates (10, 17.3)

3. Find the neutral current in the follwing.



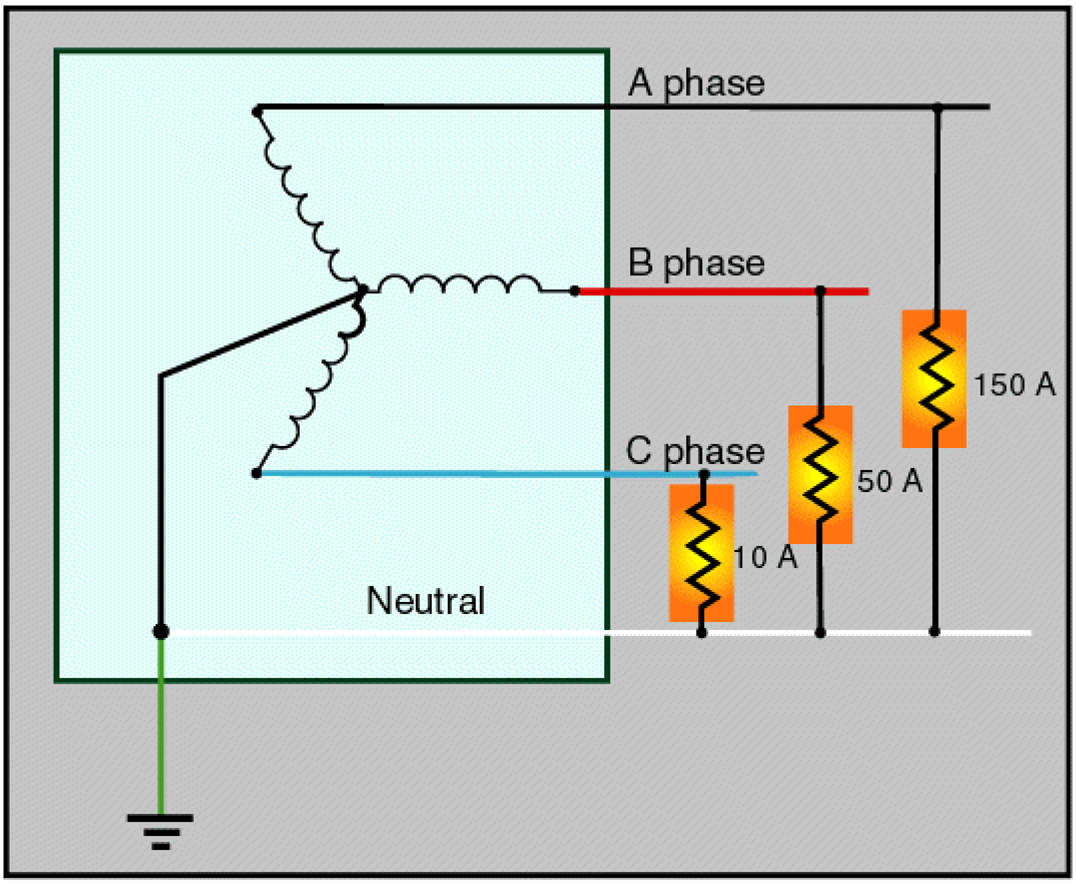
$$|A+B+C| = \sqrt{(150 \times 1 + 50 \times -.5 + 0 \times -.5)^2 + (150 \times 0 + 50 \times .866 + 0 \times -.866)^2}$$

$$|A+B+C| = \sqrt{125^2 + 43.3^2}$$

$$|A+B+C| = \sqrt{17{,}500}$$

$|A+B+C| =$ 132 amperes at coordinates (125, 43.3)

4. Find the neutral current for the following:



$$|A + B + C| = \sqrt{(150 \times 1 + 50 \times -.5 + 10 \times -.5)^2 + (150 \times 0 + 50 \times .866 + 10 \times -.866)^2}$$

$$|A + B + C| = \sqrt{120^2 + 34.6^2}$$

$$|A + B + C| = \sqrt{15{,}597}$$

$|A + B + C| =$ 125 amperes at coordinates (120, 34.6)

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Last updated: 11/10/99

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