3.8 Exponential Growth and Decay

Lots of things in nature change based on how big it is.

The math word for this
is "proportional"

The population of bunnies grows at a rate <u>proportional</u> to its population.

$$\frac{dP}{dt} = \mathrm{k}\mathrm{P}$$

k: rate
P: Population

$$\frac{1}{P}\,dP = k\,dt$$

$$\int \frac{1}{P}\,dP = \int k\,dt$$

$$\ln P = kt + c$$

$$P = e^{kt+c}$$

$$P = e^{kt}e^{c}$$

$$P = P_0 e^{kt}$$

The law of Natural Growth (if k>0) or Decay (if k<0)

$$\frac{dy}{dt} = ky$$

Exponential Functions that are solutions of (work with) $\frac{dy}{dt} = ky$

$$y(t) = y_0 e^{kt}$$

y(0) = initial value

t = time

y(t) = value after t time

k = rate of change

The rate that some particular stuff grows is proportional to how much stuff there is at that time. We start with 10 pounds of stuff, 4 hours later we have 15 pounds of stuff.
When will we have 20 pounds of stuff?

Use $y = y_0 e^{kt}$

$y_0 = 10$
$y = 15$
$t = 4$
k = what we need (rate)

$$15 = 10e^{k4}$$

$$\frac{3}{2} = e^{4k}$$

$$\ln\frac{3}{2} = 4k$$

$$\frac{\ln\frac{3}{2}}{4} = k = 0.10137$$

To find when there is 20 pounds of stuff...

Still use $y = y_0e^{kt}$

$y_0 = 10$
$y = 20$
$k = 0.10137$
t = what we need to answer

$20 = 10e^{0.10137t}$

$2 = e^{0.10137t}$

$\ln(2) = 0.10137t$

$$\frac{\ln(2)}{0.10137} = \boxed{t = 6.838\text{hours}}$$

Secret organism "Q" doubles in size every 6 hours, how long does it take to get 100 times as big?

$Q = Q_o e^{kt}$

Step 1: Find k ...

$Q_o = 1$

$Q = 2$ (doubles)

$t = 6$

k = the thing we want to find

$2 = e^{6k}$

$\ln 2 = 6k$

$$\frac{\ln 2}{6} = k$$

$0.11552 = k$

So the equation for Secret Organism Q growth is: $Q = Q_o e^{0.11552t}$

So to solve the "How long does it take to get 100 times as big" problem ...

$Q_o = 1$
$Q = 100$
$k = 0.11552$
t = the thing we want to find

$$Q = Q_o e^{0.11552t}$$

$$100 = e^{0.11552t}$$
$$\ln 100 = 0.11552t$$

$$\frac{\ln(100)}{0.11552} = t = 39.865 \text{ hours}$$

Ex. In 1950 there were 2560 million people, and in 1960 there were 3040 million folks. Assume that growth is proportional to the population.

1. Find the rate of growth.

$$P(t) = P(0)e^{kt}$$

$$P(1960) = 2560e^{10k}$$

$$3040 = 2560e^{10k}$$

$$3040 / 2560 = e^{10k}$$

$$\ln(3040 / 2560) = 10k$$

$$0.017185 = k$$

$$1.7\% \approx k$$

2. Find the population in 1993

$$P(43) = 2560e^{0.017185(43)}$$

$$P(43) \approx 5360 \text{ million}$$

Compound Interest

$$A = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

A = Value
A_0 = Initial value
r = rate
n = frequency of compounding
t = time

If continuously compounded:

$$A(t) = A_0 e^{rt}$$

...go look in your Precalc notes for how this happens...