
Ch. 9 Halftime Rundown

1. 9.1 - be cool with solving separable differential equations (the cool kids call the DiffeyQ's) - know HW 9.1.
Remember that "solving" means to find the equation from which the given differential equation comes.
2. 9.2 - Sep Diff Eq's - put in an initial condition (some point) to get a value for "C" to use with your solution
3. 9.3 - Slope Fields - these are graphs of the differential equation, and the function you sometimes graph inside them is the solution.

9.4 Euler's Method

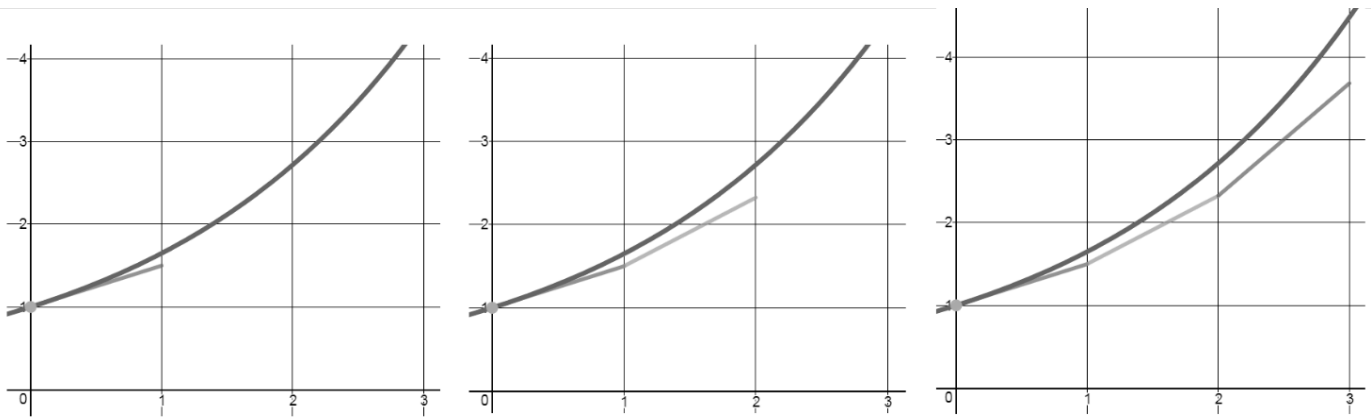
This is a way to estimate values (the same reason slope fields exist) when given a differential equation.

Basically you're going to fill out a table and choose the appropriate value, but you need to know what's going on, so that's what we're going to try to explain here.

So here's how it works. Say you know some differential equation (this means you only know how to find the slope), and some initial condition.

For a simply example, say you know the function goes through the point $(0, 1)$.

Use the slope to determine the values as you go along, and then correct the slope at particular "steps".



Here we're using step-sizes of 1, recalculating the slope at each point, and adjusting our estimate accordingly.

As you may guess, the accuracy of our estimate increases as the step-size decreases.

In the previous illustration, the graph of the function was depicted. When we need to use Euler's method, we don't know the function and so use a table to calculate estimated values of the function. Let's try an example:

$$\frac{dy}{dx} = x + 3y \quad \text{and} \quad y(2) = 3$$

Use Euler's method with a step-size of 1 to approximate $y(5)$.

x_n	y_n	y'_n
2	3	$2+3(3) = 11$
3	14	$3+3(14) = 45$
4	59	$4+3(59) = 181$
5	240	

y_n changes according to y'_n

It's not too bad when the step-size is 1. Let's try a harder one:

Use Euler's method with a step-size of 0.1 to find $y(0.4)$ if

$$\frac{dy}{dx} = x + y \quad \text{and} \quad y(0) = 1 \quad \text{Let } h = \text{step-size} = 0.1$$

x_n	y_n	$(y'_n)(h)$
0	1	$(0+1)(0.1) = 0.1$
0.1	1.1	$(0.1+1.1)(0.1) = 0.12$
0.2	1.22	$(0.2+1.22)(0.1) = 0.142$
0.3	1.362	$(0.3+1.362)(0.1) = 0.1662$
0.4	1.5282	