First, a little review:

Consider: $y = x^2 + 3$ or $y = x^2 - 5$

then: $y' = 2x$ $y' = 2x$

It doesn't matter whether the constant was 3 or -5, since when we take the derivative the constant disappears.

However, when we try to reverse the operation:

Given: $y' = 2x$ find y

$$y = x^2 + C$$

We don't know what the constant is, so we put "C" in the answer to remind us that there might have been a constant.

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If we have some more information we can find C.

Given: $y' = 2x$ and $y = 4$ when $x = 1$, find the equation for y.

(1,4)

$$y = x^2 + C$$

$$4 = 1^2 + C$$

$$3 = C$$

$$y = x^2 + 3$$

This is called an initial condition problem. We need the initial condition to find the constant.

An equation containing a derivative is called a differential equation. It becomes an initial condition problem when you are given the initial condition and asked to find the original equation.

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Solve the differential equation $\frac{dy}{dx} = \frac{\sin x}{y^2}$ knowing the equation goes through (0, 1).

$$y^2\,dy = \sin x\,dx$$

$$\int y^2\,dy = \int \sin x\,dx$$

$$\frac{y^3}{3} = -\cos x + C$$

$$\frac{1}{3} = -1 + C$$

use x = 0 and y = 1

$$\frac{4}{3} = C$$

$$\frac{y^3}{3} = -\cos x + \frac{4}{3}$$

$$y^3 = -3\cos x + 4$$

$$y = \sqrt[3]{-3\cos x + 4}$$

Solve these equations given the initial conditions:

1. $\frac{dw}{dt} = tw^2 \sin\left(t^2\right),\ w(0) = 1$

t=0
w=1

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2. $\frac{dy}{dx} = xe^y,\ y(2) = 0$

x=2
y=0

3. $\frac{dy}{dt} = \frac{t^2}{\sqrt{y}},\ y(2) = 4$

4. $\frac{dy}{dx} = x(y-2),\ y(0) = 0$

5. $\frac{dy}{dx} = x^2\sqrt{y},\ y(2) = 4$

Answers are on the next page -------------------------------------->

Answers to homework:

$$1. w = \left(\tfrac{1}{2}\cos t^2 + \tfrac{1}{2}\right)^{-1}$$

$$2. y = -\ln\left|3 - \tfrac{1}{2}x^2\right|$$

$$3. y = \left(\tfrac{1}{2}t^3 + 4\right)^{\frac{2}{3}}$$

$$4. y = 2e^{\frac{1}{2}x^2} + 2$$

$$5. y = \left(\frac{x^3+4}{6}\right)^2$$