

True or False?

10th & 12th graders
do this one:

$$\text{If } y = \sqrt[3]{x^3 + 8}$$

$$\text{then } \frac{dy}{dx} = \frac{x^2}{y^2}$$

$$\frac{dy}{dx} = \frac{x^2}{(\sqrt[3]{x^3 + 8})^2}$$

$$= \frac{x^2}{y^2}$$

Juniors do this one:

$$y = (\sqrt[3]{x^3 - e^3})^2 \quad \text{If } y = \sqrt[3]{x^3 - e^3}$$

$$\frac{dy}{dx} = \frac{x^2}{y^2}$$

$$\frac{x^2}{(\sqrt[3]{x^3 - e^3})^2} \cdot \frac{x^2}{(\sqrt[3]{x^3 - e^3})^2} = \frac{x^2}{y^2}$$

$$x^{2/3} = \sqrt[3]{x^2} = \sqrt[3]{x^2}$$

Some terms:

Differential equation - an equation representing the relationship between a derivative and its independent variable.

$$\text{Ex: } \frac{dy}{dx} = 3x^2$$

Separable differential equation - A differential equation that can be re-written in the form $N(y) dy = M(x) dx$

Solutions to separable differential equations are simply the original function y from which the differential equation is derived.

We find the solution by integrating both sides of a differential equation, once its separated.

$$\cancel{dx} \cdot \frac{dy}{dx} = y^2 x dx$$

$$y^{-2} dy = x dx$$

$$\int y^{-2} dy = \int x dx$$

$$-y^{-1} = \frac{x^2}{2} + c$$

$$-y^{-1} = \frac{x^2}{2} + c$$

$$y^{-1} = -1 \left(\frac{x^2}{2} + c \right)$$

$$y^{-1} = -\frac{x^2}{2} + c$$

$$y^{-1} = -\frac{x^2 + 2c}{2}$$

$$y = -\frac{2}{x^2 + 5} + 5$$

$$\frac{dy}{dx} = x^2 y$$

$$y^{-1} dy = x^2 dx$$

$$\int \frac{1}{y} dy = \int x^2 dx$$

$$\ln|y| = \frac{x^3}{3} + c$$

$$\ln|y| = \frac{x^3}{3} + c$$

$$e^{\ln|y|} = e^{\frac{x^3}{3} + c}$$

$$y = \pm e^{\frac{x^3}{3} + c}$$

$$x^{2+3} = x^2 x^3$$

$$y = \pm e^{\frac{x^3}{3}} e^c$$

$$y = \pm c e^{\frac{x^3}{3}}$$

$$y = c e^{\frac{x^3}{3}}$$

