Opener - do on sheet-halved o' paper (SHOP) and turn in

Find the area between the curves

$y = x^2$ and $y = 8 - x^2$

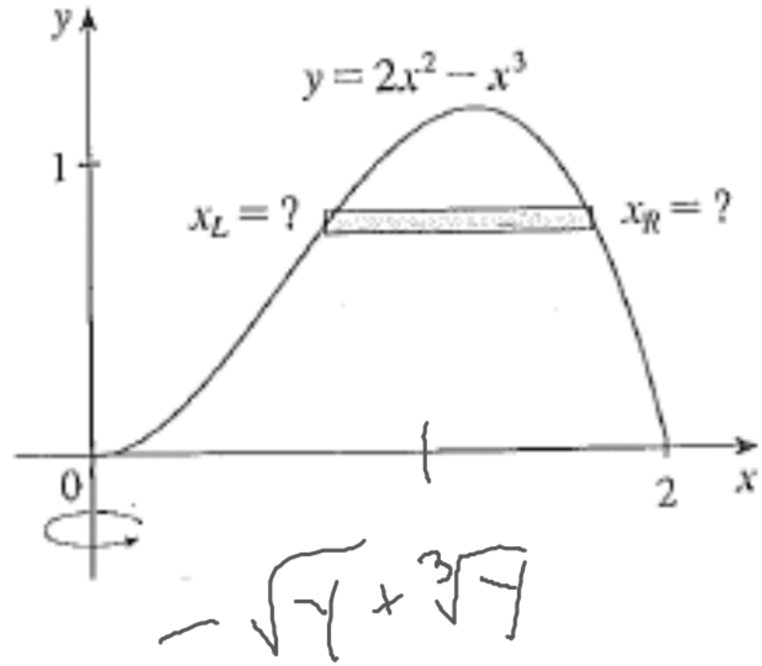
64/3

Hey! If you know what this is then you can use integration to find volumes by cylindrical shells!!!!!!

6.3: Volumes by Cylindrical Shells

Let's start by reviewing washers and disks with the function $y = 2x^2 - x^3$, rotated about the y-axis.

Sketch a typical washer and identify the pertinent radii.

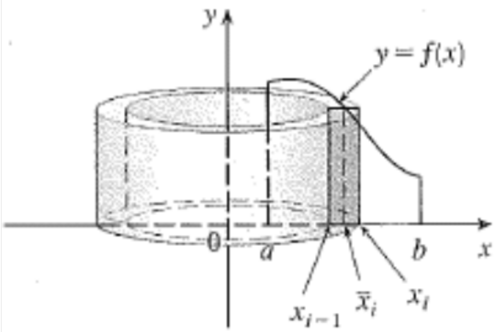


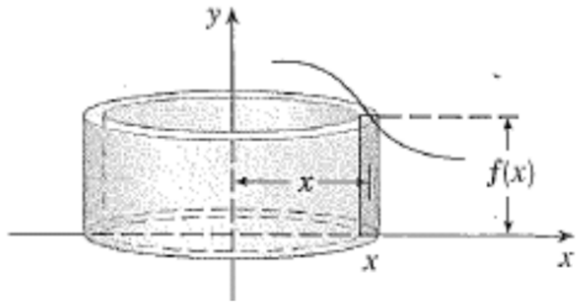
Among other headaches, you'd need the cubic formula (a degree beyond the quadratic formula). Thanks, but no thanks.

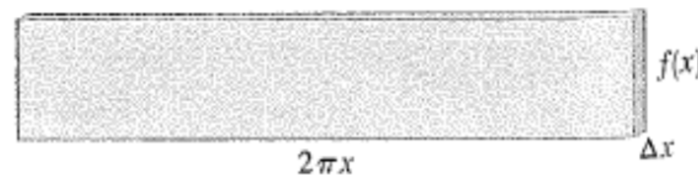
$$x = \sqrt[3]{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)+\sqrt{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)^2+\left(\frac{c}{3a}-\frac{b^2}{9a^2}\right)^3}}$$

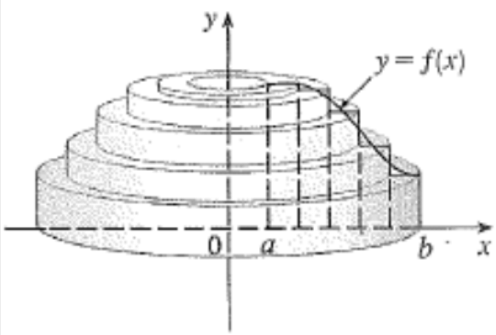
$$+ \sqrt[3]{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)-\sqrt{\left(\frac{-b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)^2+\left(\frac{c}{3a}-\frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}.$$

In order to take care of this problem, we can use "shells" instead of washers, as pictured below. We are cutting the volume perpendicular to the x-axis instead of the y-axis.





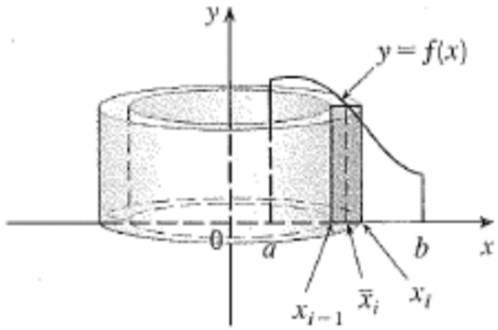




$$V = 2\pi rh\,\Delta r \qquad \boxed{1}$$

and it can be remembered as

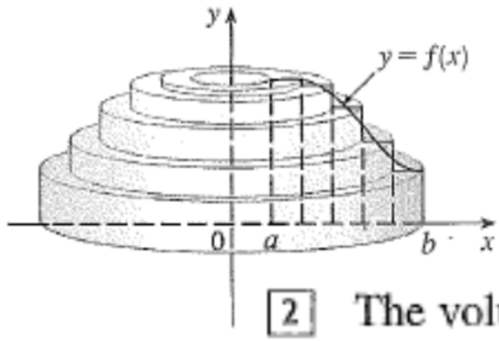
$$V = [\text{circumference}][\text{height}][\text{thickness}]$$



We go from the Riemann Sum to the integral: $\lim_{n\to\infty}\sum_{i=1}^{n} 2\pi\bar{x}_i f(\bar{x}_i)\,\Delta x = \int_a^b 2\pi x f(x)\,dx$

Where the integral means:

$$\int_a^b \underbrace{(2\pi x)}_{\text{circumference}}\ \underbrace{[f(x)]}_{\text{height}}\ \underbrace{dx}_{\text{thickness}}$$

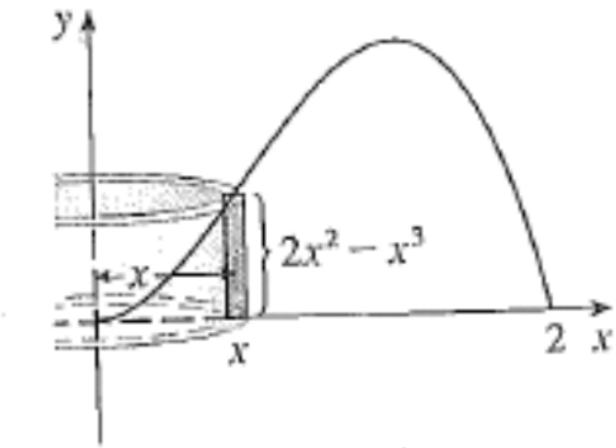


Definition: In general for vertical shells

2 The volume of the solid in Figure 3, obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b, is

$$V = \int_a^b 2\pi x f(x)\,dx \qquad \text{where } 0 \leqslant a < b$$

Example 1: Let's go back to $y = 2x^2 - x^3$ and solve for the volume bound by the function as we rotate around the y-axis.



Recall your formula and what we are doing here:

$$\int_a^b \underbrace{(2\pi x)}_{\text{circumference}} \underbrace{[f(x)]}_{\text{height}} \underbrace{dx}_{\text{thickness}}$$

$$V = \int_0^2 2\pi x\left(2x^2 - x^3\right)dx$$

$$V = 2\pi\int_0^2 \left(2x^3 - x^4\right)dx$$

$$V = 2\pi\left[\tfrac{1}{2}x^4 - \tfrac{1}{5}x^5\right]_0^2$$

$$V = 2\pi\left(8 - \tfrac{32}{5}\right)$$

$$V = \tfrac{16}{5}\pi$$