

## 6.1 - Areas Between Curves

Just what you think it would be, but you typically see some nastier integrals to deal with here. Try this one, and then we'll go over how they make it nasty.

Determine the area between the curves

$$y = 1 + \sqrt{x}$$

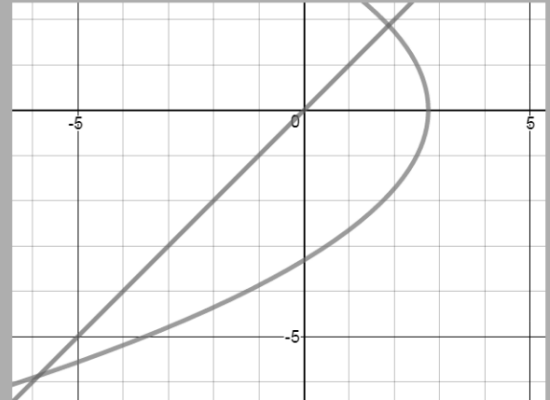
$$y = \frac{(3+x)}{3}$$

Use a graphing calculator!

Find the area between  $4x + y^2 = 12$  and  $y = x$

You could probably do this with "vertical rectangles" like we usually do, that is, with respect to  $x$ .

Instead, just to get you used to it, let's do it horizontally, with respect to  $y$ .



The first part is to find the points of intersection. Using substitution:

$$4x + x^2 = 12$$

$$x^2 + 4x - 12 = 0$$

$$(x + 6)(x - 2) = 0$$

$$x = -6 \text{ and } x = 2$$

so,  $y = -6$  and  $y = 2$

(since  $y = x$ )

Integrating with respect to  $y$  means both functions have to be solved for  $x$ , so use

$$x = -\frac{1}{4}y^2 + 3$$

"higher function - lower function"

$$\begin{aligned} & \int_{-6}^2 \left[ \left( -\frac{1}{4}y^2 + 3 \right) - y \right] dy \\ &= \left[ -\frac{1}{12}y^3 + 3y - \frac{1}{2}y^2 \right]_{-6}^2 \\ &= \left( -\frac{2}{3} + 6 - 2 \right) - \left( 18 - 18 - 18 \right) \\ &= -\frac{2}{3} + 22 = \frac{64}{3} \end{aligned}$$