The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a,b]$, then the function

$$F(x)=\int_a^x f(t)\,dt$$

has a derivative at every point in $[a,b]$, and

$$\frac{dF}{dx}=\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$$

How to think of FTC part 1:

So $\int f(x)\,dx$ undoes a $\frac{d}{\mathrm{dx}} f\left(x\right)$

And $\frac{d}{\mathrm{dx}} f\left(x\right)$ undoes a $\int f(x)\,dx$.

Just like $\sin(\arcsin(x)) =$

And $\frac{3x}{3} =$

And $x + 5 - 5 =$

And $\sqrt{x^2} =$

Let's do one we know how to do:

$$\frac{d}{dx}\int\left(4x^2 - 2x\right)dx$$

First, evaluate the antiderivative of the guts of this function.

$$\frac{4}{3}x^3 - x^2$$

Ok, now we should take the derivative of that function.

$$\frac{\cancel{3}\cdot 4}{\cancel{3}}x^2 - 2x$$

What do we notice?!

Let's try this out, and don't over think it.

$$\frac{d}{dx}\int_0^x \frac{1}{1+t^2}\,dt = \frac{1}{1+x^2}$$

You might want to overthink this one:

HINT: derivative of the guts

$$\frac{d}{dx}\int_7^{3x^2} \sin(t)\,dt = 6x\sin\left(3x^2\right)$$

The Fundamental Theorem of Calculus, Part 2

If f is continuous at every point of $[a,b]$, and if F is any antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

(Also called the **Integral Evaluation Theorem**)

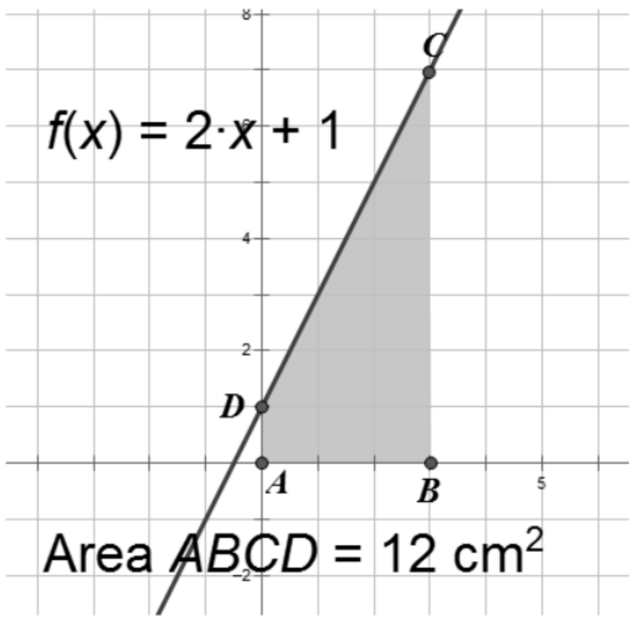
FTC part 2 allows us to view the integral in 2 ways:

1. We can look at the integral as an "area under the curve" function on (a,b).
2. Or we can look at the integral as "the displacement of an antiderivative on (a,b)".

Let's take a look at these two interpretations:

Area and Displacement

Area interpretaion

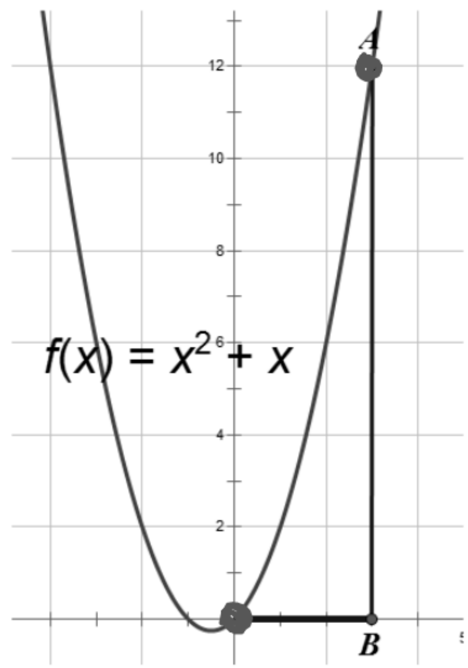


Here is a function:
f(x)=2x+1 on (0,3)

$$\int_0^3 (2x+1)\mathrm{dx}$$

Displacement, we need ANY antiderivative

$$F(x) = x^2 + x$$



$$F(3) - F(0) =$$

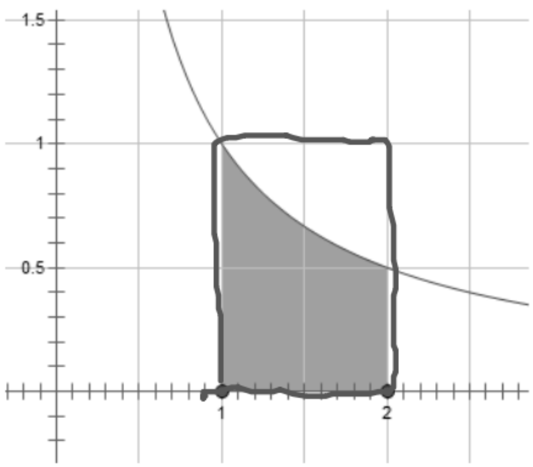
$$12 - 0 = \boxed{12}$$

[redacted]

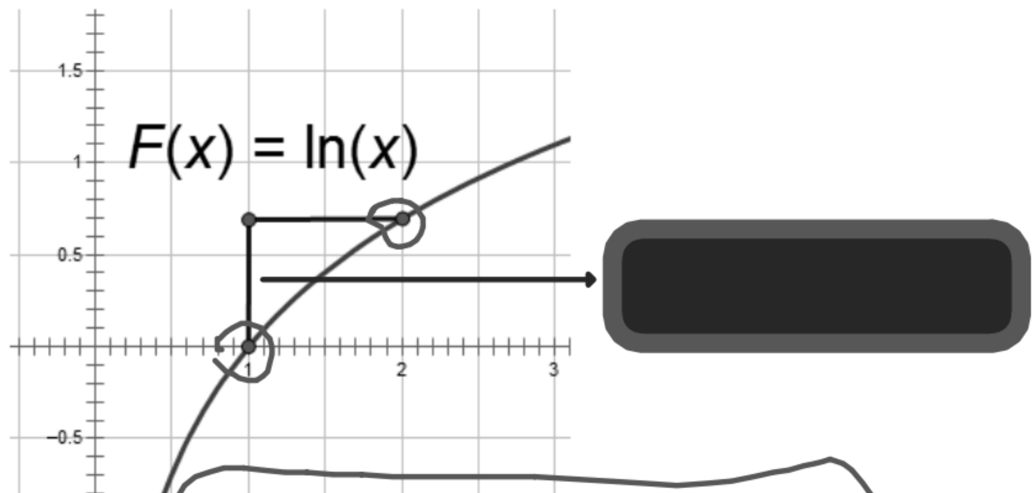
Let's take a look at these two interpretations:

Area and Displacement

Area interpretaion



Displacement, we need ANY antiderivative



$\ln 2 - \ln 1$

Here is a function:
f(x)=1/x on (1,2)
or: $\int_1^2 \frac{1}{x} dx$

Evaluating definite integrals

One easy way to integrate polynomials is use "up a power, over the power":

ex. $= \int_0^2 (y-1)(2y+1)\,dy$

$= \int_0^2 \left(2y^2 - y - 1\right) dy$ Make a single polynomial

$= \frac{2}{3}y^3 - \frac{1}{2}y^2 - y \,\Big|_0^2$ Up a power, over a power

$= \left(\frac{2}{3}2^3 - \frac{1}{2}2^2 - 2\right) - \left(\frac{2}{3}0^3 - \frac{1}{2}0^2 - 0\right)$ F(2) - F(0)

$= \boxed{\frac{4}{3}}$ Math

① $\int_0^5 \frac{5}{9x}\,dx$

$\int \alpha x\,dx =$

$\alpha \int x\,dx$

$\frac{5}{9}\int_0^5 \frac{1}{x}\,dx$