4.7 - Optimizing Problems

This section is all about find the most/least whatever, and it's the word-problem version of finding maximums/minimums.

The key is this: maximums and mimimums occur when a derivative is zero.

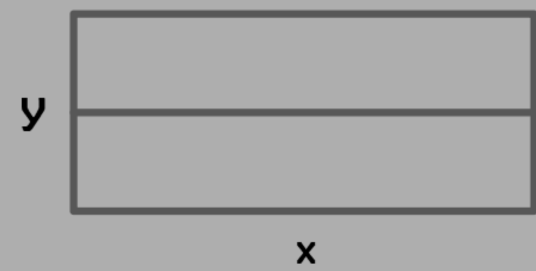
There're a series of steps you might want to check out on p. 322 if you're into that sort of thing.

Also, there are example problems in the book that reflect some of the homework types.

We'll do one example as a family. Family. That's a funny word if you say it a bunch of times in a row.

A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can we do this so as to minimize the cost of the fence?

1. Draw a picture:



2. Write two equations: $xy = 1{,}500{,}000$

   $p = 3x + 2y$

3. Equation to be optimized (max or min): $p = 3x + 2y$

4. Solve OTHER equation for a variable: $y = (1{,}500{,}000)/x$

5. Substitute into the optimization equation:

p = 3x + 2y
p = 3x + (3,000,000)/x

6. Take the derivative of optimization equation:

$$p' = 3 - \frac{3{,}000{,}000}{x^2}$$

$$p' = \frac{3x^2 - 3{,}000{,}000}{x^2}$$

7. Set optimized equation's derivative equal to zero:

$$0 = \frac{3x^2 - 3{,}000{,}000}{x^2}$$

8. Solve for the variable:

$$0 = \frac{3x^2 - 3,000,000}{x^2}$$

$$0 = 3x^2 - 3,000,000$$

$$1,000,000 = x^2$$

$$1,000 = x$$

You should really check this:

p' < 0 when x < 1000 so p is decreasing BEFORE the critical point

p' > 0 when x > 1000 so p is increasing AFTER the critical point,

then p(1000) must be a minimum.

Knowing that x should be 1000, we can find the other dimension using the equation $y = (1{,}500{,}000)/x$

$$y = (1{,}500{,}000)/1000$$

$$y = 1500$$

The farmer should set up the field 1000 feet by 1500 feet, with the middle fence parallel to the short side of the fence.