Consider:

$$\lim_{x \to 1} \frac{lnx}{x - 1}$$

We run into two problems with limits:

1) indeterminate form of type $\frac{0}{0}$

2) indeterminate form of $\frac{\infty}{\infty}$

4.4: L'Hospitals's Rule: (Pronounced Low-pee-tall)

Suppose f and g are differentiable on an open interval *I*, except possibly at the number *a* in *I* and that for all $x \neq a$ in *I*, g'(*x*) ≠ 0. If we have an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Example:

Let's take another look at $\lim\limits_{x\to 1}\dfrac{\ln x}{x-1}$

$$\lim_{x\to 1}\frac{\ln x}{x-1} \overset{\text{H}}{=} \lim_{x\to 1}\frac{\frac{1}{x}}{1} = 1$$

Use this equal sign with the "aitch" on top to signify the use of l'Hospital's rule.

Sometime you have to be crafty and use the rule twice:

$$\lim_{x \to \infty} \frac{x + x^2}{1 - 2x^2} \overset{H}{=} \lim_{x \to \infty} \frac{1 + 2x}{-4x}$$

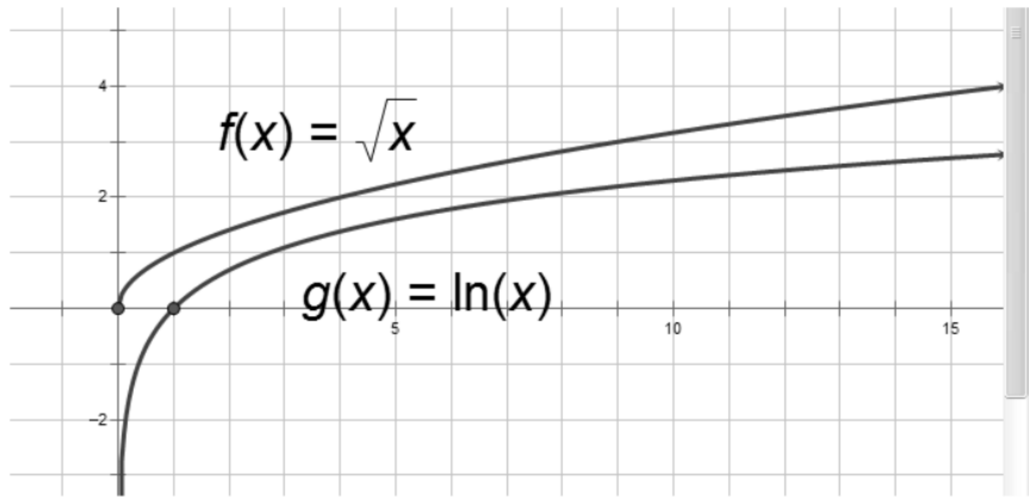
We've still got $\frac{\infty}{\infty}$

Problem 15 from 4.4: To begin, what type of indeterminate limit do we have?

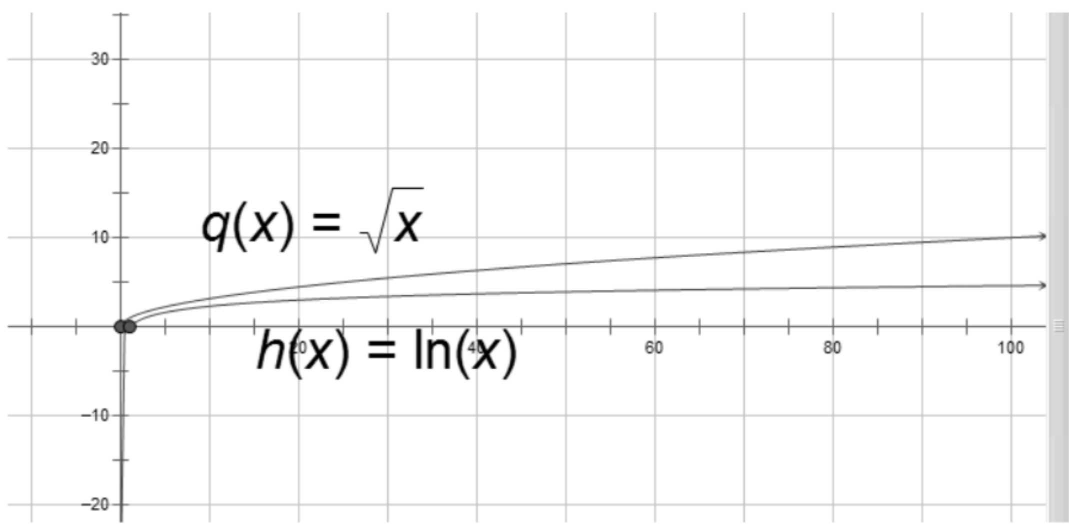
$$\lim_{x \to \infty} \frac{lnx}{\sqrt{x}}$$

Take a guess on this answer given the graphs of each function...

$$\lim_{x \to \infty} \frac{lnx}{\sqrt{x}}$$



What appears to happen for their "end behavior"?



Evaluate the function by applying L'Hospital's Rule.

$$\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$$

$$\frac{\frac{1}{x}}{\frac{1}{2}x^{-1/2}} = \frac{0}{\sqrt{\infty}} = 0$$