Do Now - Halloween '17

Kindly put your calculators away.

Consider the function: $f(x)=3x^4-4x^3-12x^2 + 8$

Find the critical points of the function.

$f(x)=3x^4-4x^3-12x^2 + 8$

Increasing and decreasing

If a function f is continuous and differentiable, then

- Where $f'(x) > 0$, then f is increasing
- Where $f'(x) < 0$, then f is decreasing

f'(x)=12x(x-2)(x+1), giving critical points at -1, 0 and 2.

Use the <u>First Derivative Test</u> to determine the intervals of increase and decrease on f.

Interval	12x	x-2	x+1	f '(x)	f
x<-1					
-1<x<0					
0<x<2					
x>2					

Locations of Relative Extrema

Relative extrema (max or min) occur at **critical points**.

If f'(x) is negative from the left, and positive from the right of a critical point, there is a minimum (f(x) is changing from decreasing to increasing).

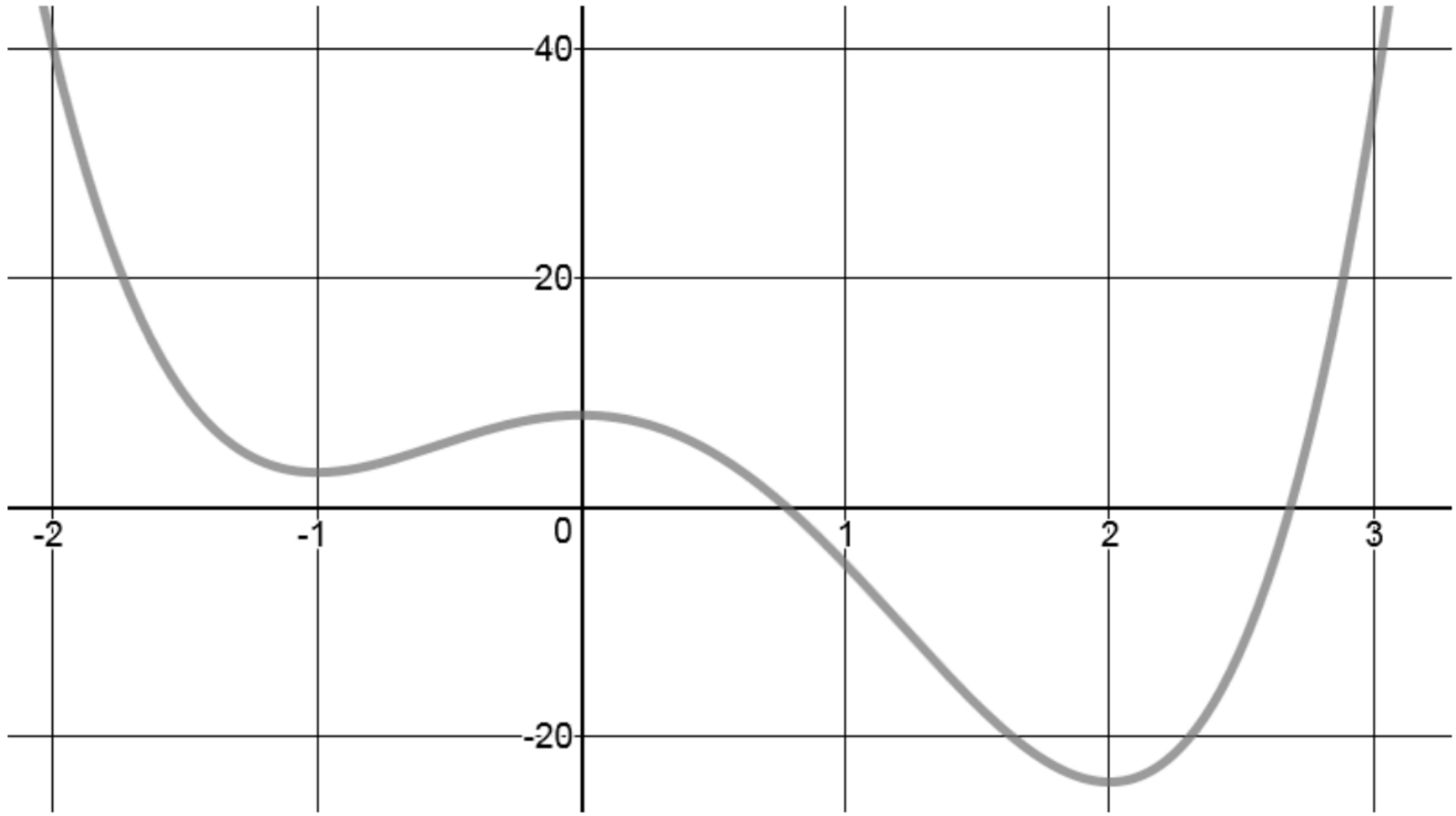
If f'(x) is positive from the left, and negative from the right of a critical point, there is a maximum (f(x) is changing from decreasing to increasing).

If f'(x) has the same sign on both sides of a critical point, then f(x) does not have a relative extremum at that critical point.

$f(x) = 3x^4-4x^3-12x^2 + 8$

$f'(x) = 12x^3 - 12x^2 - 24x$

Critical points: x = -1 x = 0, x = 2



Relative min at x = -1 and x = 2, relative max at x = 0

Definition:

Concavity

If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward** on I.

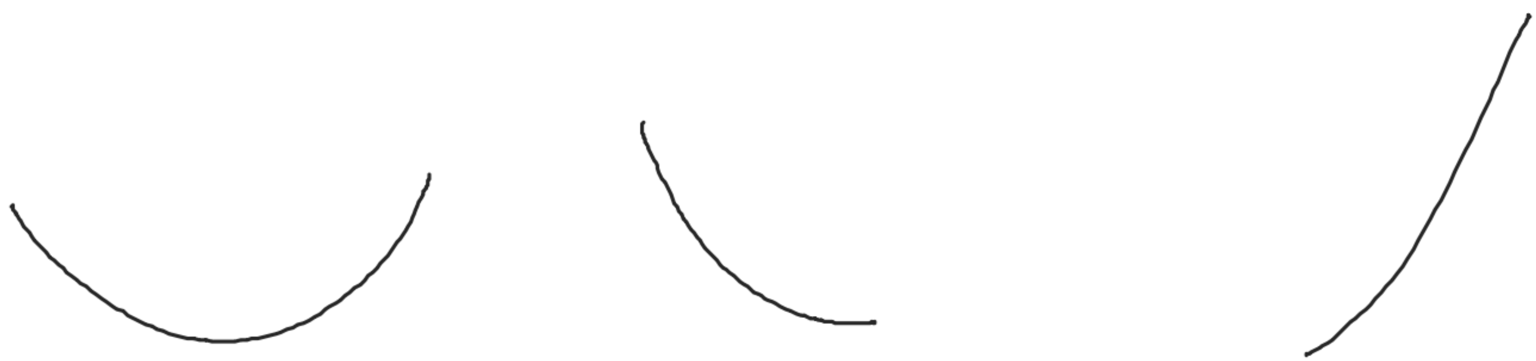
Definition:

Concavity

Is the dent on the top part or the bottom part?

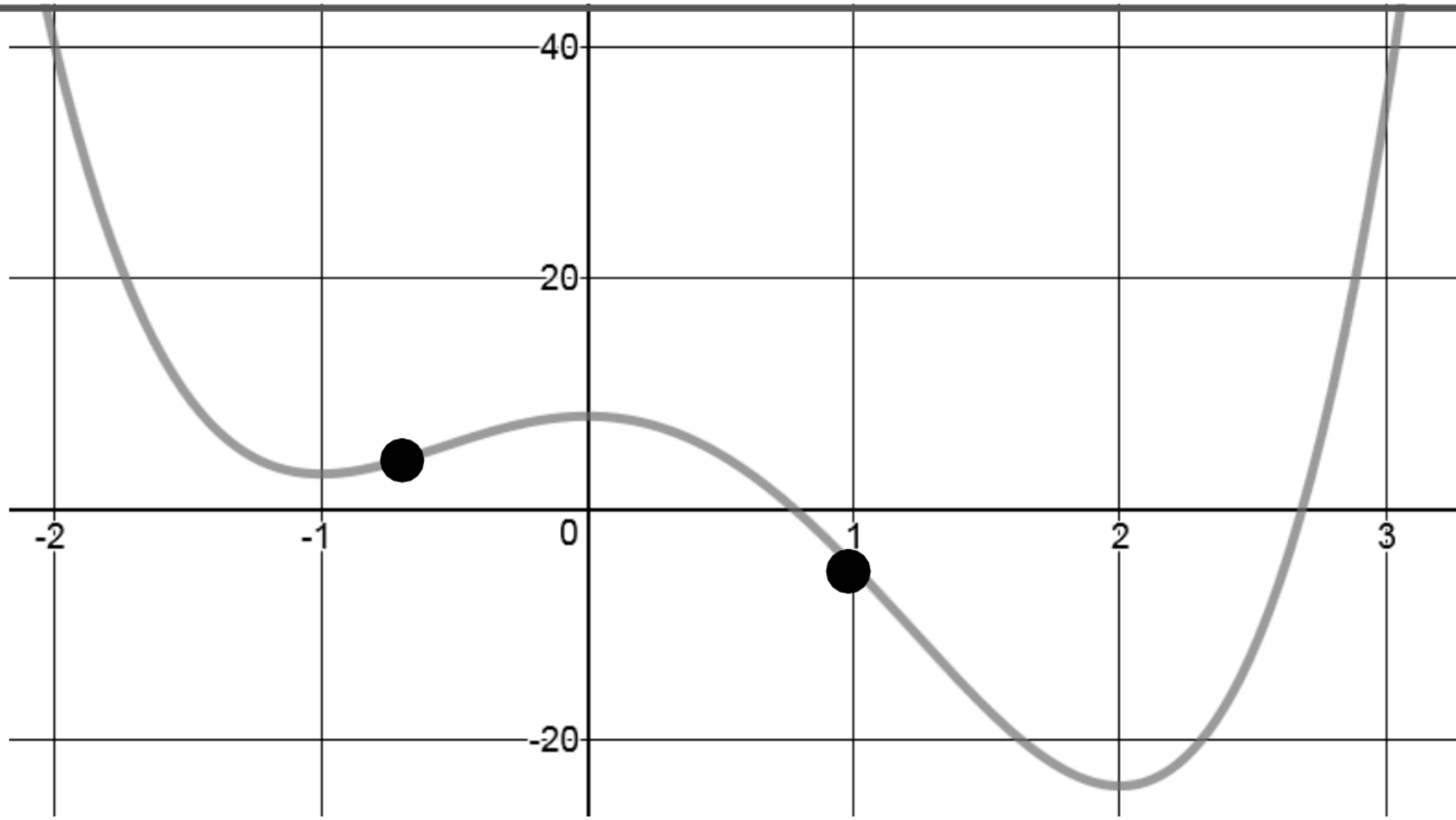
A function is concave down if it looks like this on an interval:

A function is concave up if it looks like this:



Inflection points - can only occur when $f''(x) = 0$ or is undefined.

These are like critical points, but using the second derivative. Inflection points are where concavity changes.



Using our original function

$f(x) = 3x^4 - 2x^3 - 12x^2 + 8$

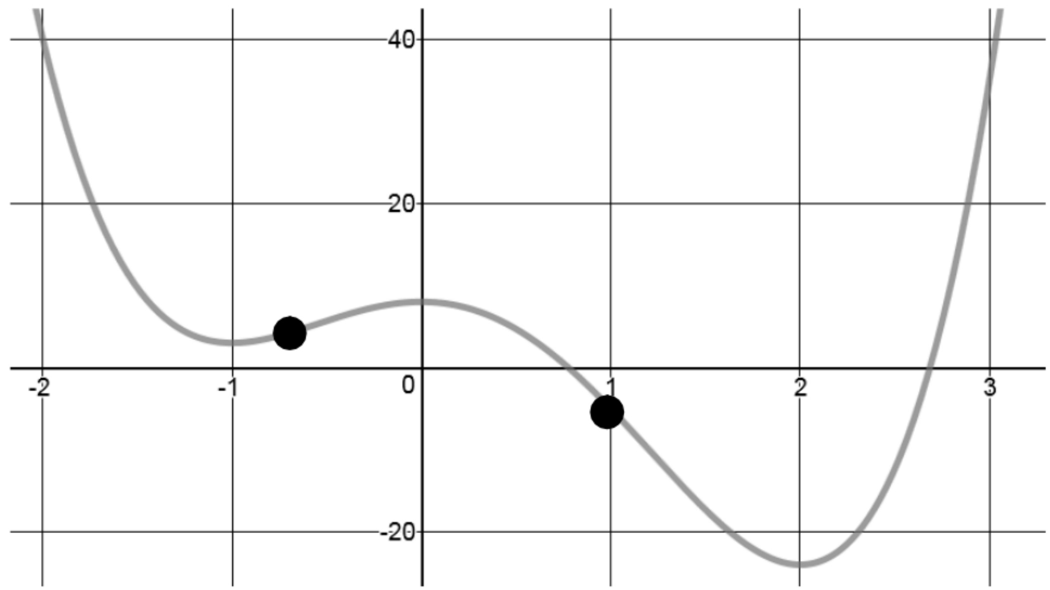
$f'(x) = 12x^3 - 6x^2 - 24x$

$f''(x) = 36x^2 - 12x - 24$

$= 12(3x^2 - x - 2)$

$0 = 12(3x + 2)(x - 1)$ set $f''(x) = 0$

$x = -2/3$ or $x = 1$



If a function f is twice differentiable (you can take the derivative twice) at $x = c$, then the graph of f is

- concave upward at $(c, f(c))$ if $f''(c) > 0$
- concave downward at $(c, f(c))$ if $f''(c) < 0$

f''(x)=12(3x + 2)(x - 1), giving possible inflection points at $x = -2/3$ and $x = 1$.

Use the Second Derivative Test to determine the intervals of concavity on f.

Interval	3x+2	x-1	f ''(x)	f
x<-2/3				
-2/3<x<1				
x>1				

Here, try this:

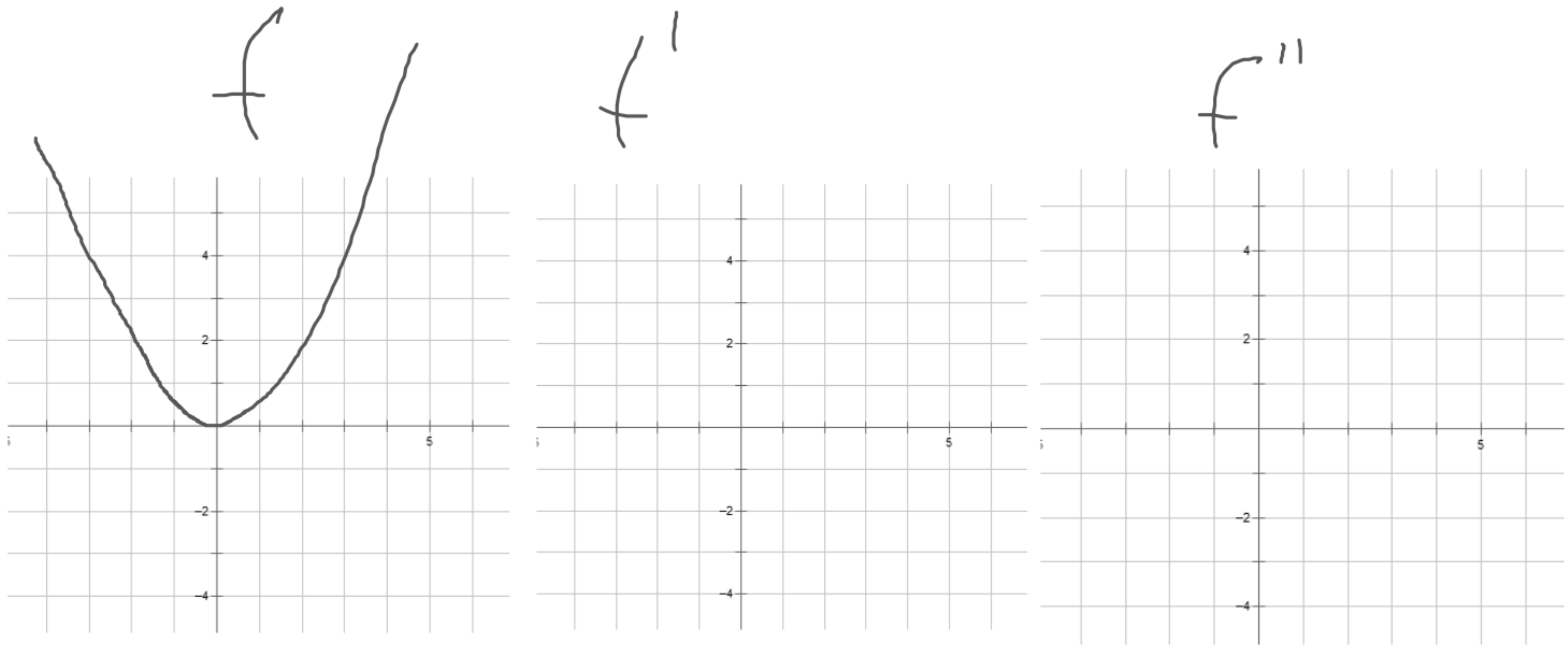
Determine for the function f(x):

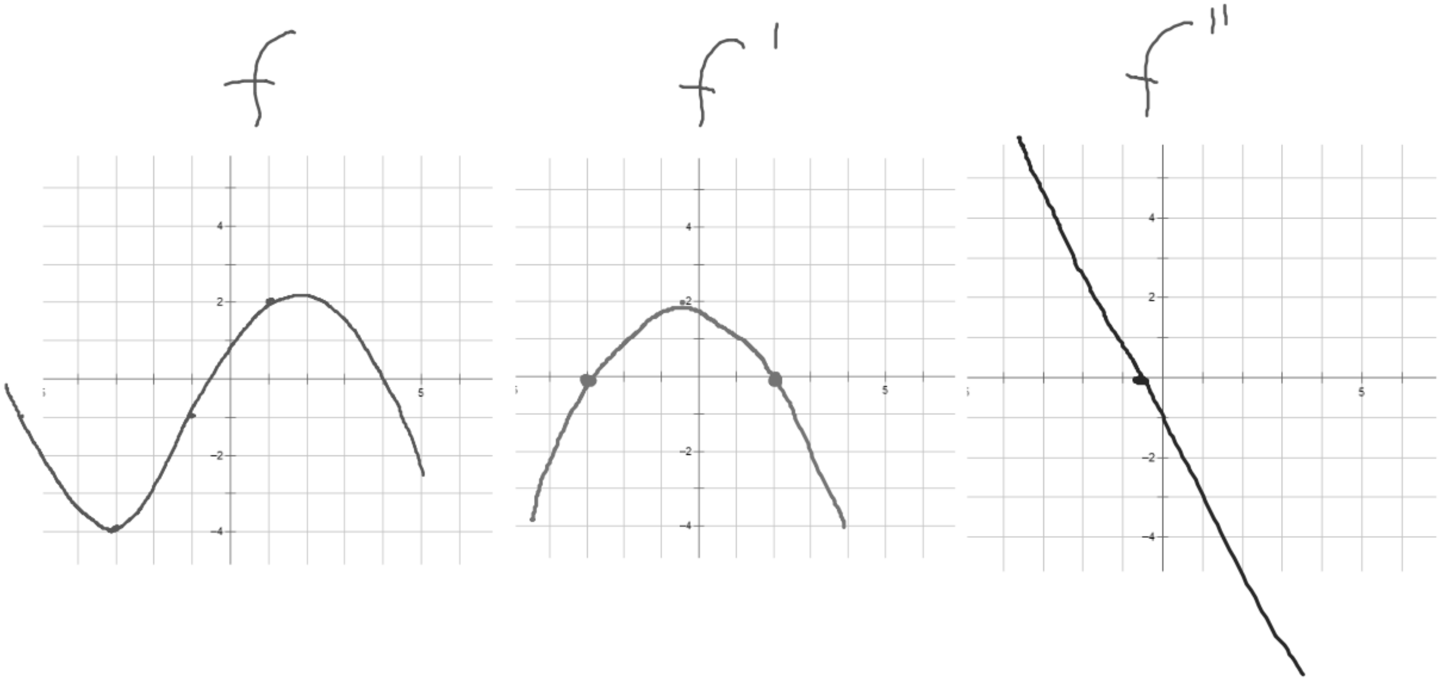
1. The intervals of increase and decrease
2. Locations of local maximums and minimums
3. The intervals of concavity
4. The location of inflection points

$$f(x) = 4x^3 + 3x^2 - 6x + 1$$

In general:

If f"(x)>0 for all x in I, then the graph of f is concave upward on I.
If f"(x)<0 for all x in I, then the graph of f is concave down on I.





Second Derivative Test:

If f'(c)=0 and f"(c)>0, then f has a ____________________at c.

If f'(c)=0 and f"(c)<0, then f has a ____________________at c.

For Example:

Problem 23 on p.295

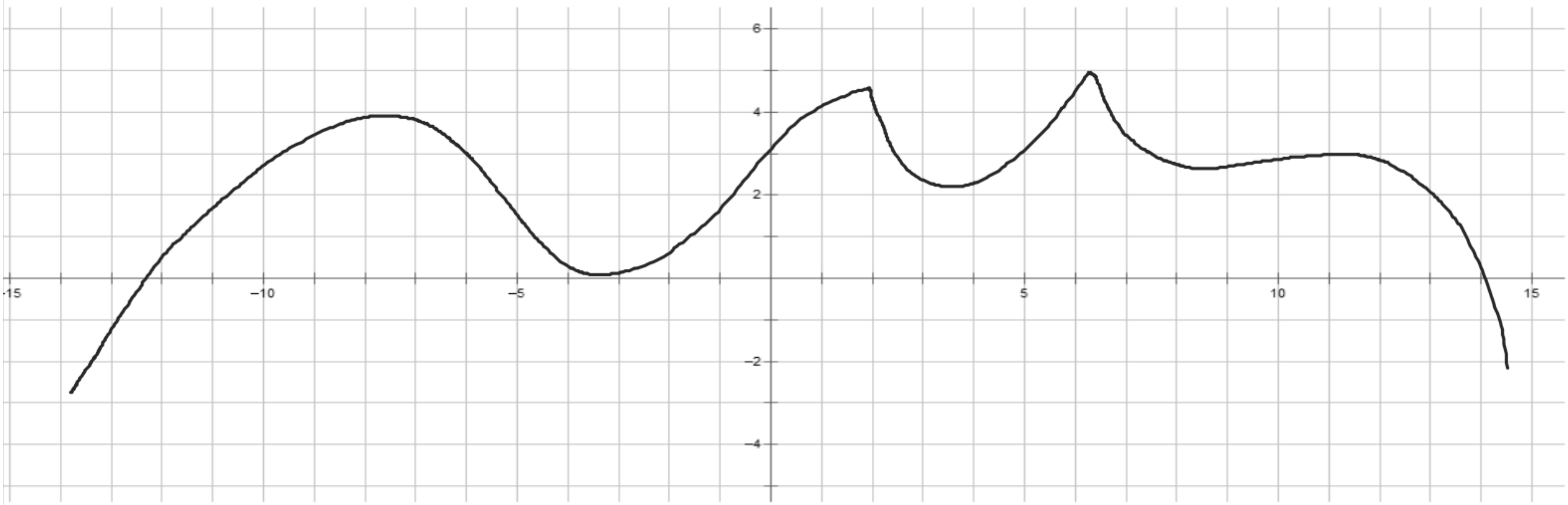
Suppose f" is continuous on All Real Numbers

a) If $f'(2) = 0$ and $f''(2) = -5$, what can you say about f?

b) If $f'(6) = 0$ and $f''(6) = 0$, what can you say about f?

Definition:

Inflection Point: A point P on a curve y=f(x) is called an inflection point if f is continuous there and the curve changes from concave up to concave down or from concave down to concave up at P.



Where are the changes in concavity?