Plot the following points: f(-2) = 5 and f(4) = -3.

Sketch some random graph connecting these points.

What is the average rate of change from x = 2 to x = 6?

Draw a secant line connecting x = 2 and x = 6.

Does it appear that **your graph** has a **derivative** that equals the **slope** of your **secant line**? If so, **mark** where this happens.

Great, this is all the Mean Value Theorem (MVT) says.

MVT: let f be a function that satisfies the following hypotheses:

1) f is continuous on [a,b]
2) f is differentiable on (a,b)

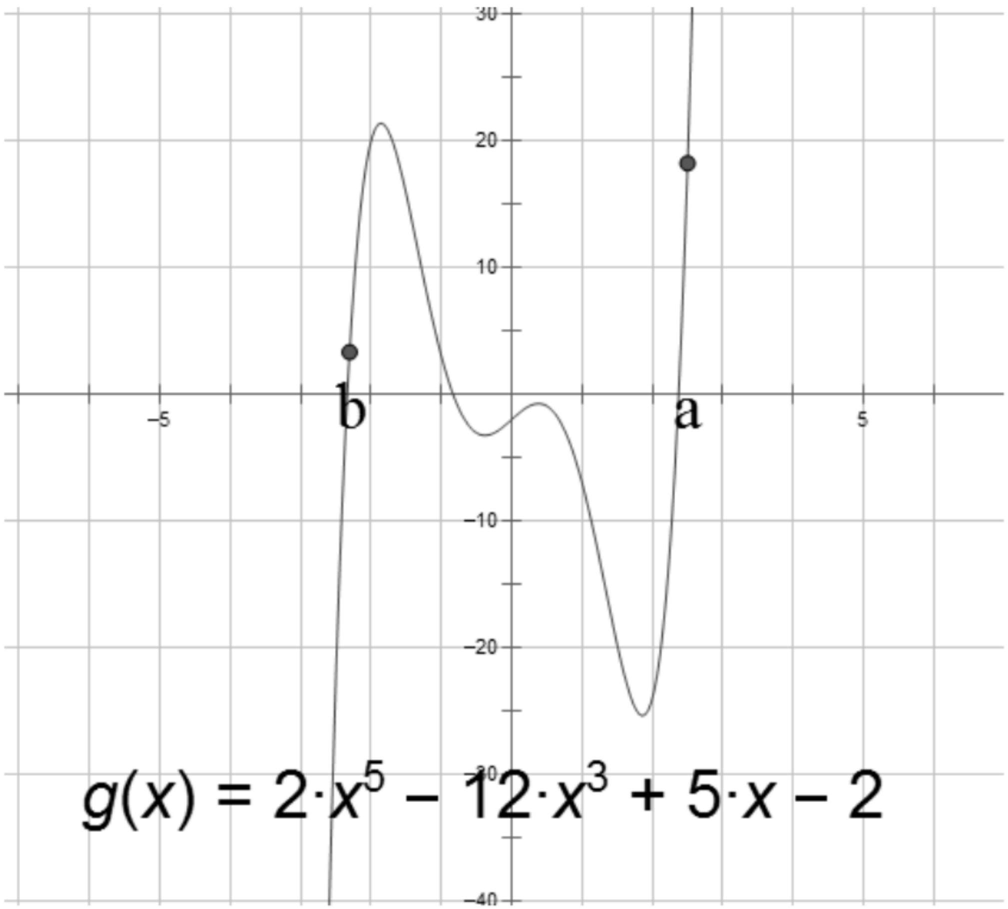
Then there is a number c in (a,b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or equivalently:

$$f'(c)(b - a) = f(b) - f(a)$$

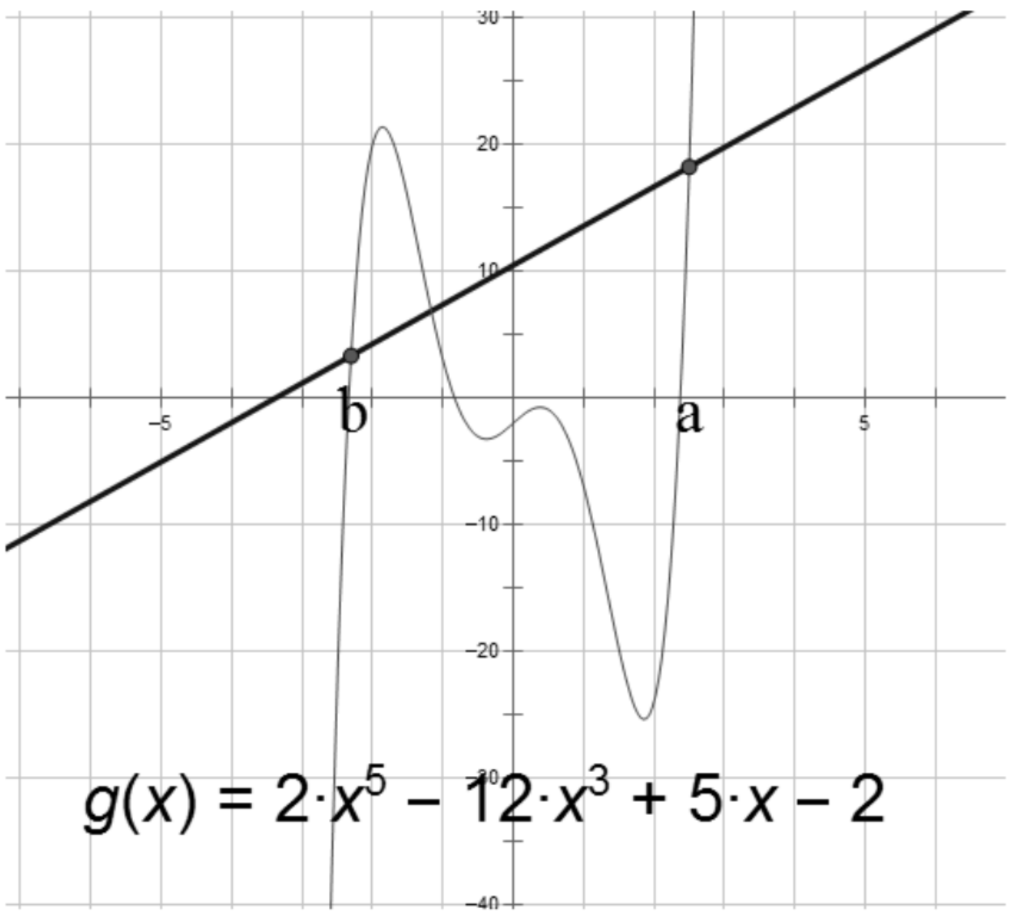
For example:



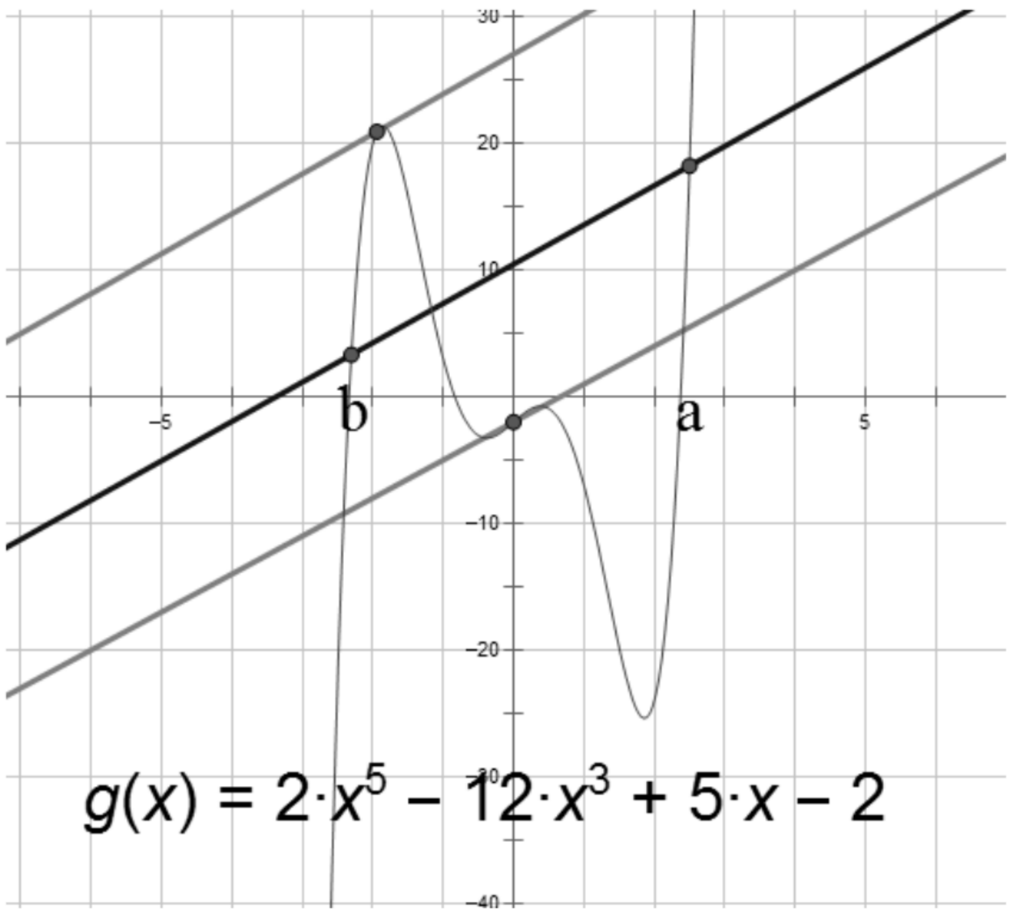
Next, Sketch the average rate of change on your interval [a,b]. APPROXIMATE it.

Remember, the average rate of change is just slope.

Next, find a value c on your domain of f, such that f'(c) is equal to the slope of the secant line you just constructed.



When we say f'(c) is the same as the slope of your tangent line, all we are saying is that the slope of the tangent line is equal to that of the secant line.

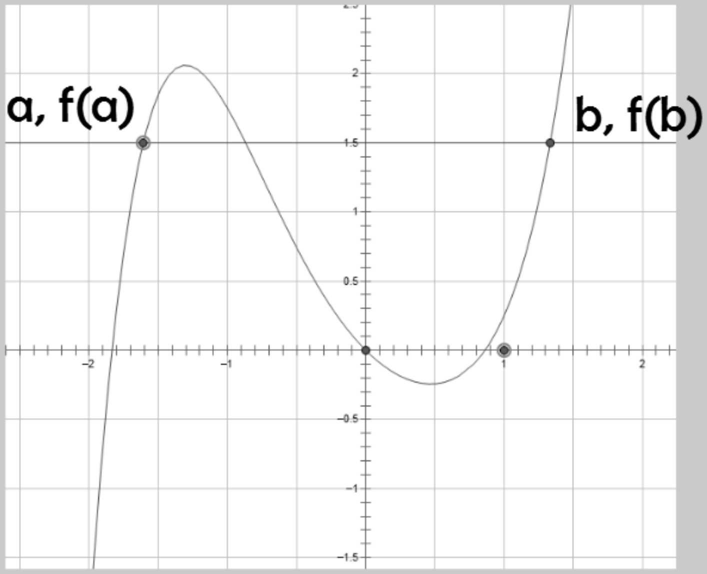


I have constructed 2 in green here. How many more do there appear to be?

A more simple version of the Mean Value Theorem is Rolle's Theorem, which says that if

$f(x)$ is continuous on $[a, b]$
$f(c)$ is differentiable on (a, b)
$f(a) = f(b)$

then there is a number c in (a, b) such that $f'(c) = 0$.



Rolle's Theorem says there must be a point on the curve f(c) where the derivative is zero.

Can you find them?

There are two places!

Example problem:

Find a value c on [0,2] that satisfies the MVT, for the function: $f(x)=x^3-x$

What is this even saying? There is an average value (the slope) of the points 0, f(0) and 2, f(2), and somewhere between x = 0 and x = 2 there exists an instantaneos slope with this same average value.

1. Find the slope of the (secant) line that connects x = 0 and x = 2.

   f(0) = 0   f(2) = 6 $m = \frac{6-0}{2-0} = 3$

2. Now find where on the function this instantaneous slope exists.

   $f(c)=c^3-c$
   $f'(c)=3c^2-1 = 3$

   $c = \frac{2}{\pm\sqrt{3}}$

   We'll just use $c = \frac{2}{\sqrt{3}}$ since we need c to be in the interval [0, 2].