4.1 - Maximum and Minimum Values

Even though the graphing calculator and the computer have eliminated the need to routinely use calculus to graph by hand and to find maximum and minimum values of functions, we still study the methods to increase our understanding of functions and the mathematics involved.

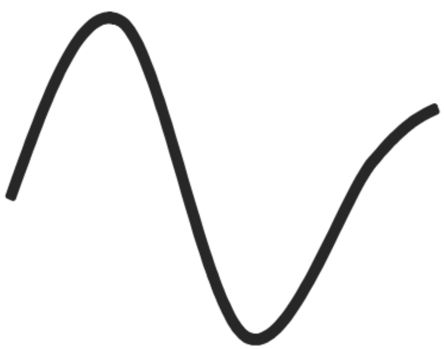
Absolute extreme values are either maximum or minimum points on a curve.

They are also sometimes called absolute extrema.

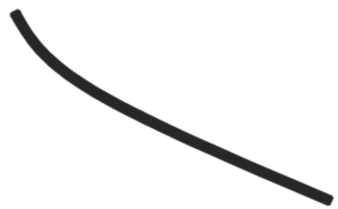
(*Extrema* is the plural of the Latin *extremum*.)

Extreme Value Theorem:

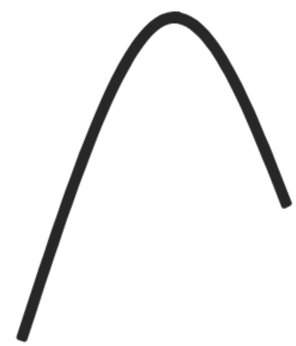
If f is continuous over a CLOSED interval, then f has a maximum and minimum value over that interval.



Maximum & minimum at interior points

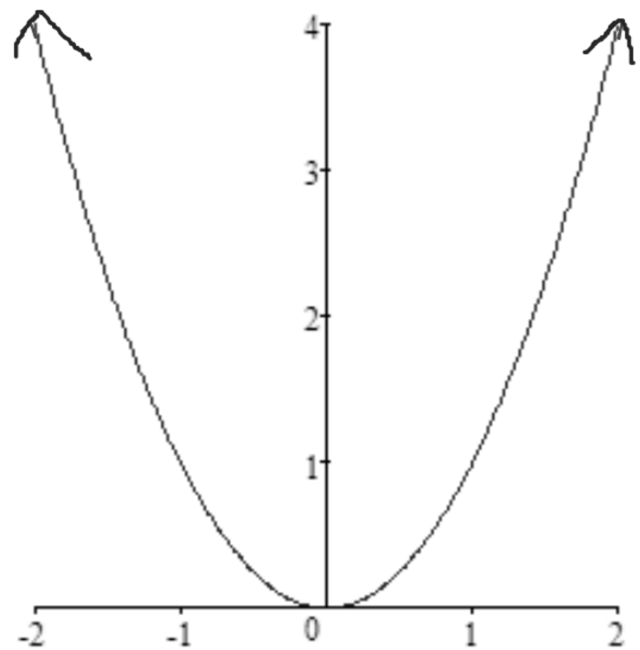


Maximum & minimum at endpoints



Maximum at interior point, minimum at endpoint

Extreme values can be in the <u>interior</u> or the <u>end points</u> of a function.



No Absolute Maximum

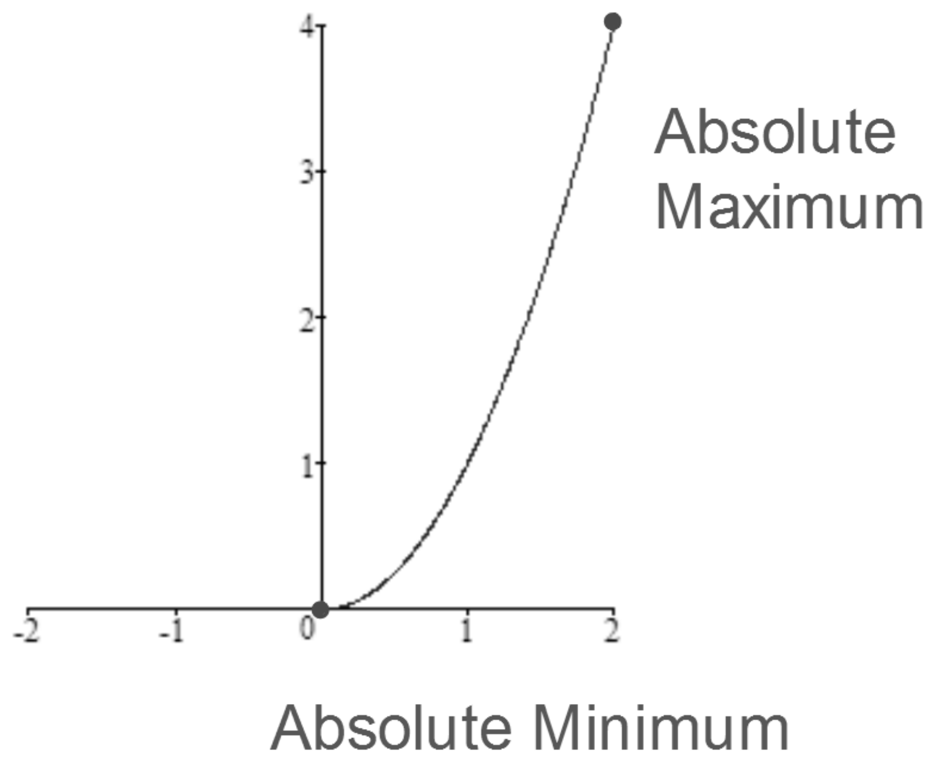
$$y = x^2$$

$$D = (-\infty, \infty)$$

Absolute Minimum

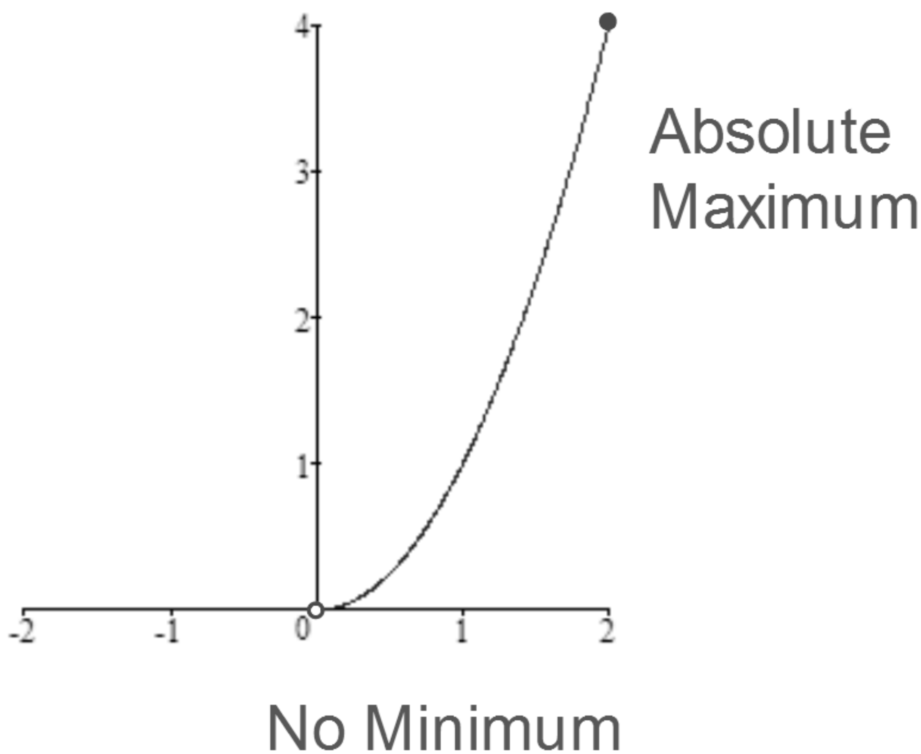
$$y = x^2$$

$$D = [0, 2]$$



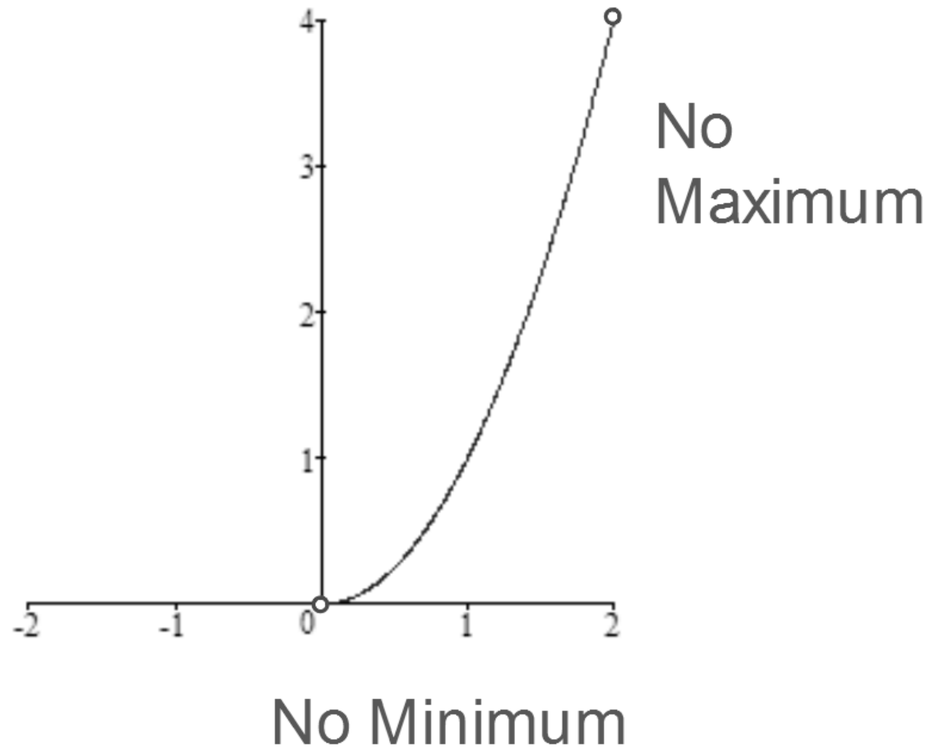
$$y = x^2$$

$$D = (0, 2]$$



$$y = x^2$$

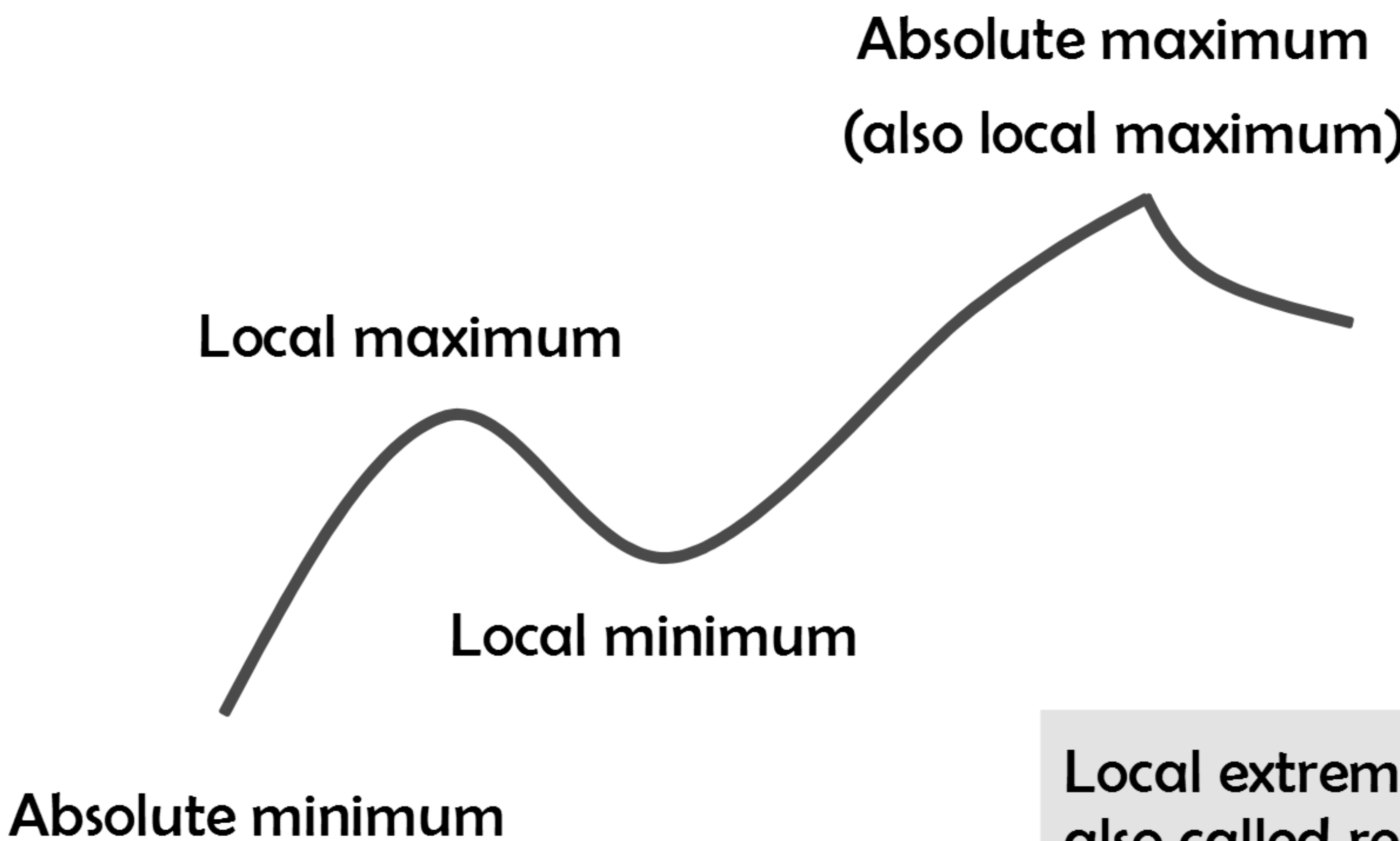
$$D = (0, 2)$$



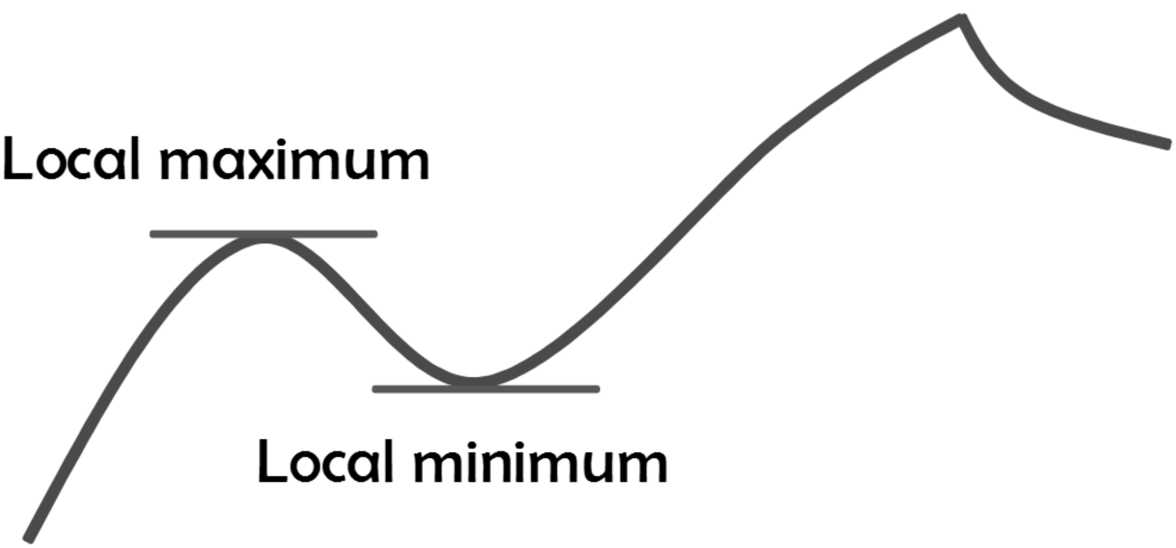
Local Extreme Values:

A local maximum is the maximum value within some open interval.

A local minimum is the minimum value within some open interval.



Local extremes are also called relative extremes.



Notice that local extremes in the interior of the function occur where f' is zero or f' is undefined.

Local Extreme Values:

If a function f has a local maximum value or a local minimum value at an interior point c of its domain, and if f' exists at c, then

$$f'(c) = 0$$

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Critical Point:

A point in the domain of a function f at which $f'=0$ or f' does not exist is a **critical point** of f.

Note:

Maximum and minimum points in the interior of a function always occur at critical points, but critical points are not always maximum or minimum values.

Ex. FINDING ABSOLUTE EXTREMA

Find the absolute maximum and minimum values of $f(x) = x^3 - 3x^2 + 1$ on the interval $-\frac{1}{2} \le x \le 4$.

Find critical numbers:
$f'(x) = 3x^2 - 6x = 3x(x - 2)$
$f'(x) = 0$ when $x = 0$ or $x = 2$

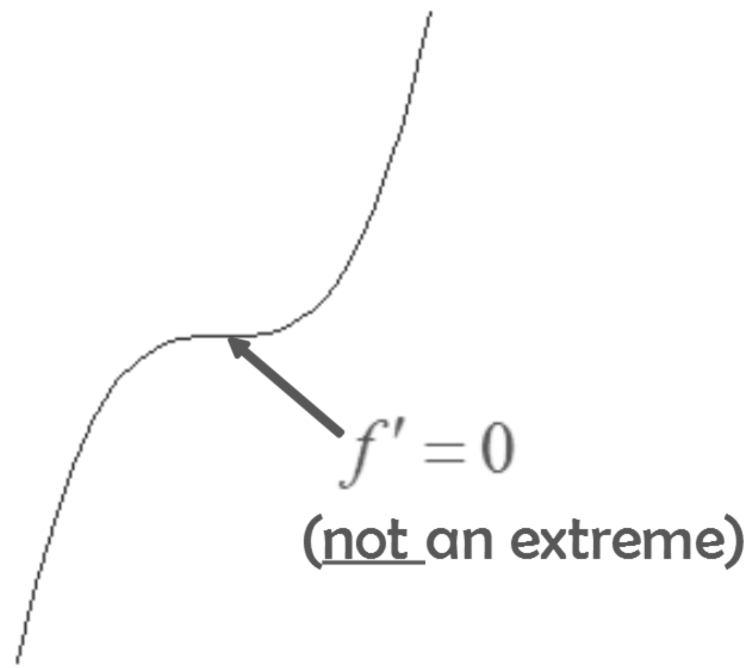
Each of these are in our interval, so try them:
$f(0) = 1$ $f(2) = -3$

Try the endpoints:
$f(-1/2) = 1/8$ $f(4) = 17$

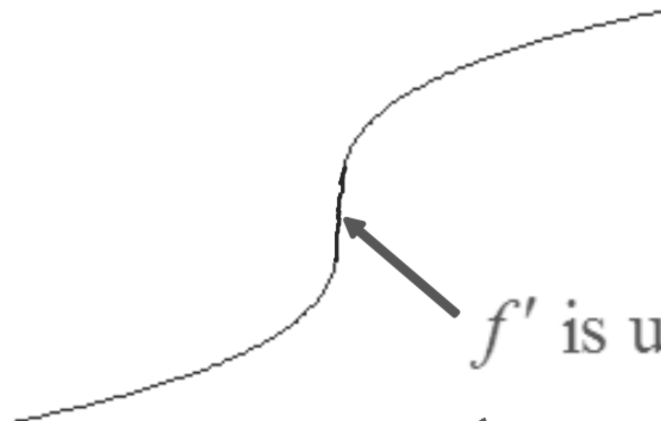
The lowest is the absolute minimum, so $f(2) = -3$ is the absolute minin
The highest is the absolute maximum, so $f(4) = 17$ is the absolute max

Critical points are <u>not always</u> extremes!

$$y = x^3$$



$y = x^{1/3}$



f' is undefined.

(<u>not</u> an extreme)

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