Related Rates

[redacted]

Take this formula:

$$A = lw$$

$$\frac{dA}{dt} = L \cdot \frac{dw}{dt} + W \frac{dL}{dt}$$

Say we know that the length grows at a rate of 8 ft/sec when the length is 4 feet, and the area is 24 feet and growing at a rate of 60 ft^2/sec.

What is the rate that the width increases?

$\frac{dw}{dt}$ FIND!

$\frac{dA}{dt} = 60 \text{ FT}^2/\text{sec}$ $\quad L = 4 \text{ FT}$ $\quad W = \frac{A}{L} = \frac{24}{4} = 6 \text{ FT}$ $\quad \frac{dL}{dt} = 8 \text{ FT/sec}$

$$60 = 4 \cdot \frac{dw}{dt} + 6 \cdot 8$$

$$-48 \qquad\qquad -48$$

$$\frac{12}{4} = \frac{4 \frac{dw}{dt}}{4}$$

$$3 \text{ FT/sec} = \frac{dw}{dt}$$

3.9 - Related Rates

This is all about word problems. If you blow air into a balloon, the volume is changing, but so is the radius.

We're going to make an equation for the situation, and then differentiate it (usually with respect to time).

Air is being pumped into a balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius increasing when the diameter is 50 cm?

1. Given: the volume increases (dV/dt) at a rate of = 100 cm^3/s.
2. Unknown: the rate the radius increases (dr/dt) when r = 25 cm.

$$V = \frac{4}{3}\pi r^3$$

Sometimes you need to "reckon" the formula

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Differentiate; don't put the r = 25 in until the end!

$$\frac{100}{4\pi r^2} = \frac{dr}{dt}$$

Solve for dr/dt

$$\frac{100}{4\pi 25^2} = \frac{dr}{dt}$$

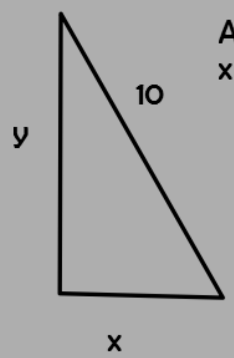
$$\frac{1}{25\pi}\ \text{cm} / s = \frac{dr}{dt}$$

$$0.013\ \text{cm/s} \approx \frac{dr}{dt}$$

Either of these are fine answers.

Just be sure that if you answer as a 'decimal' that it's accurate to THREE places!

A 10 ft ladder rests against a wall. The bottom slides away at 1 ft/s. How fast is the ladder sliding down the wall when the bottom is 6 ft from the wall?



Again, don't use $x = 6$ until the end!

$$\frac{dx}{dt} = 1\,\frac{\text{ft}}{\text{sec}}$$

Find $\frac{dy}{dt}$ when $x = 6\text{ft}$

$$x^2 + y^2 = 100$$ "Reckon" an equation for the situation

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$ Differentiate with respect to time

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$ Solve for whachu whachu want.

Now we have x and dx/dt, but we also need y.
Use the original equation to find y.

$$y = \sqrt{100 - 6^2} = 8$$

$$\frac{dy}{dt} = -\frac{6}{8}(1)$$

$$\frac{dy}{dt} = -0.75 \text{ ft/s}$$

The ladder is sliding down the wall at a rate of 0.75 ft/s

I didn't need the negative sign since I said "down"