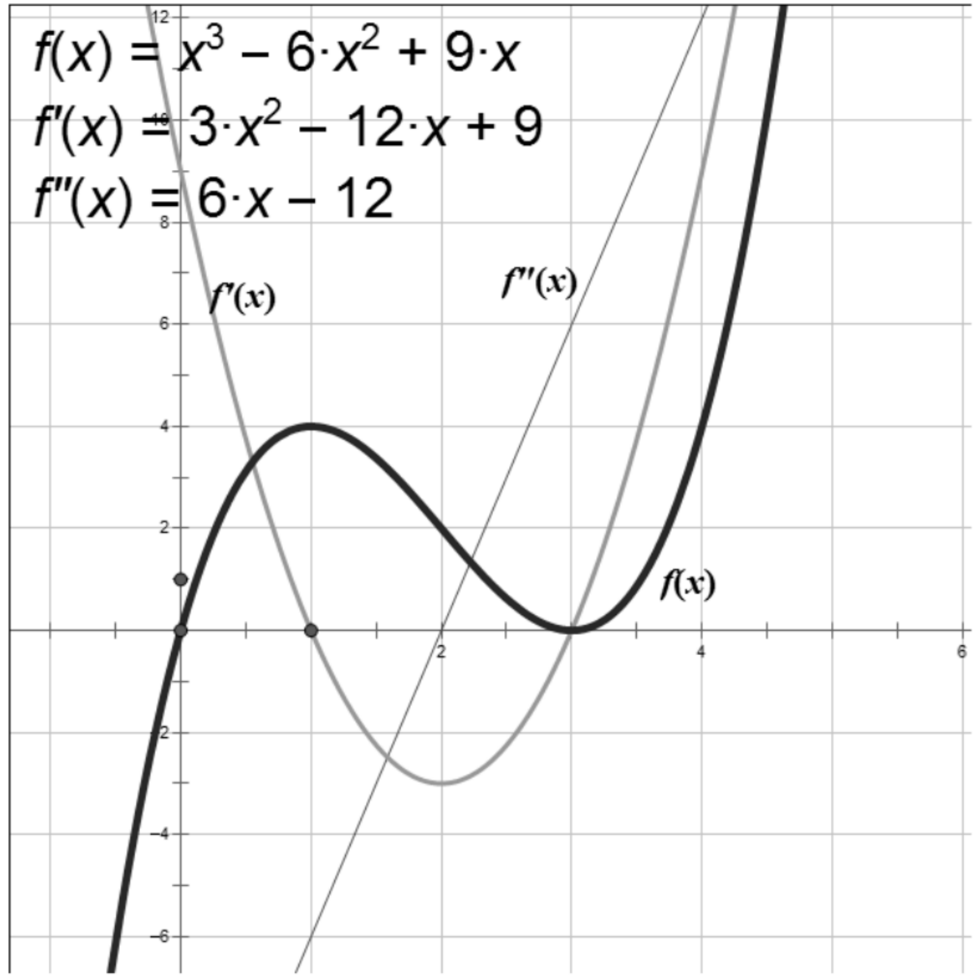
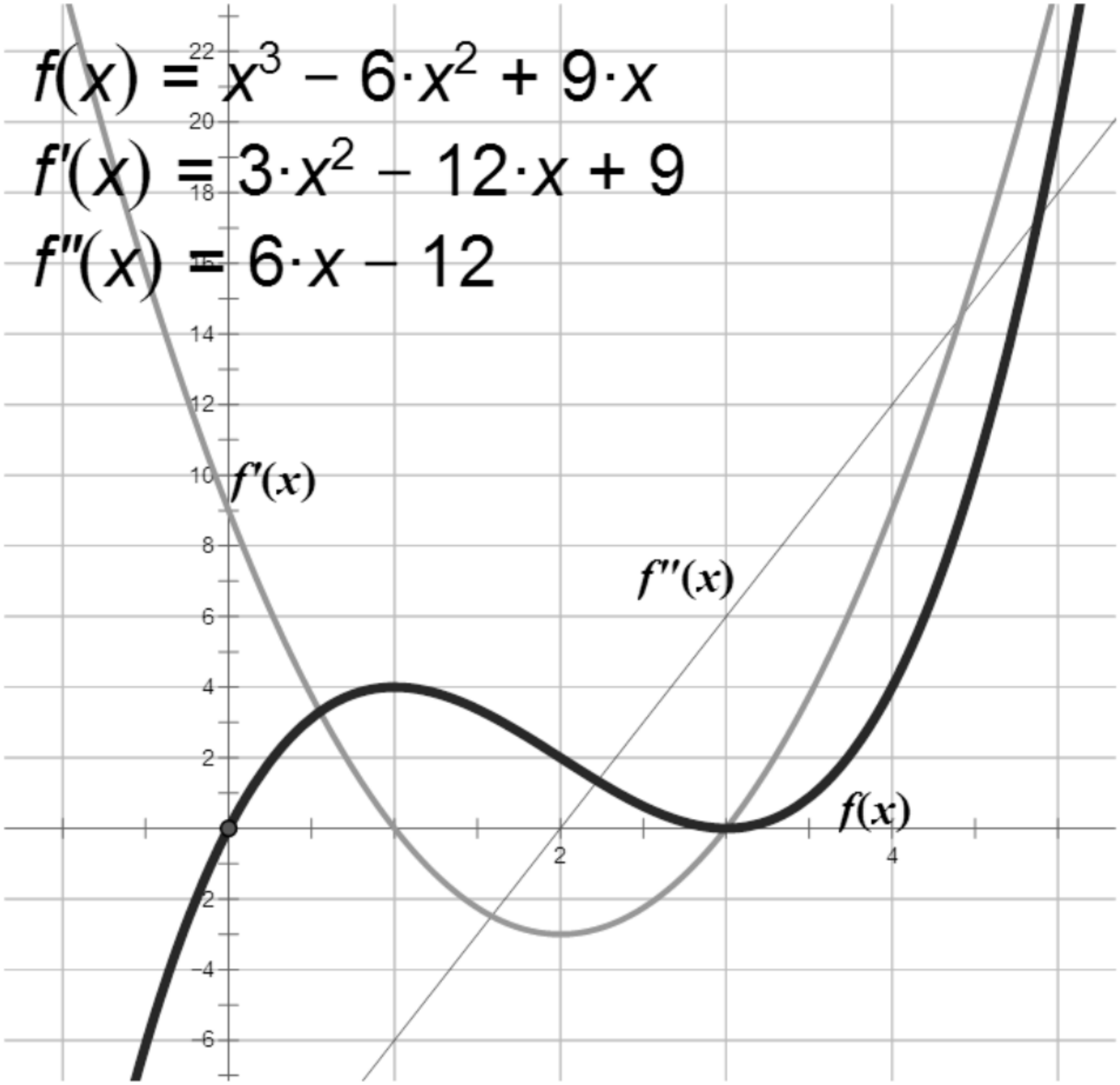
3.4 Rates of change in the natural and social sciences

The position (s) of a particle is given by the equation f(x), where x is measured in feet and y represents seconds.

Velocity (v) is given by f'(x), and is measured as feet/sec.

Acceleration (a) is f"(x), and is measured as feet/sec^2.



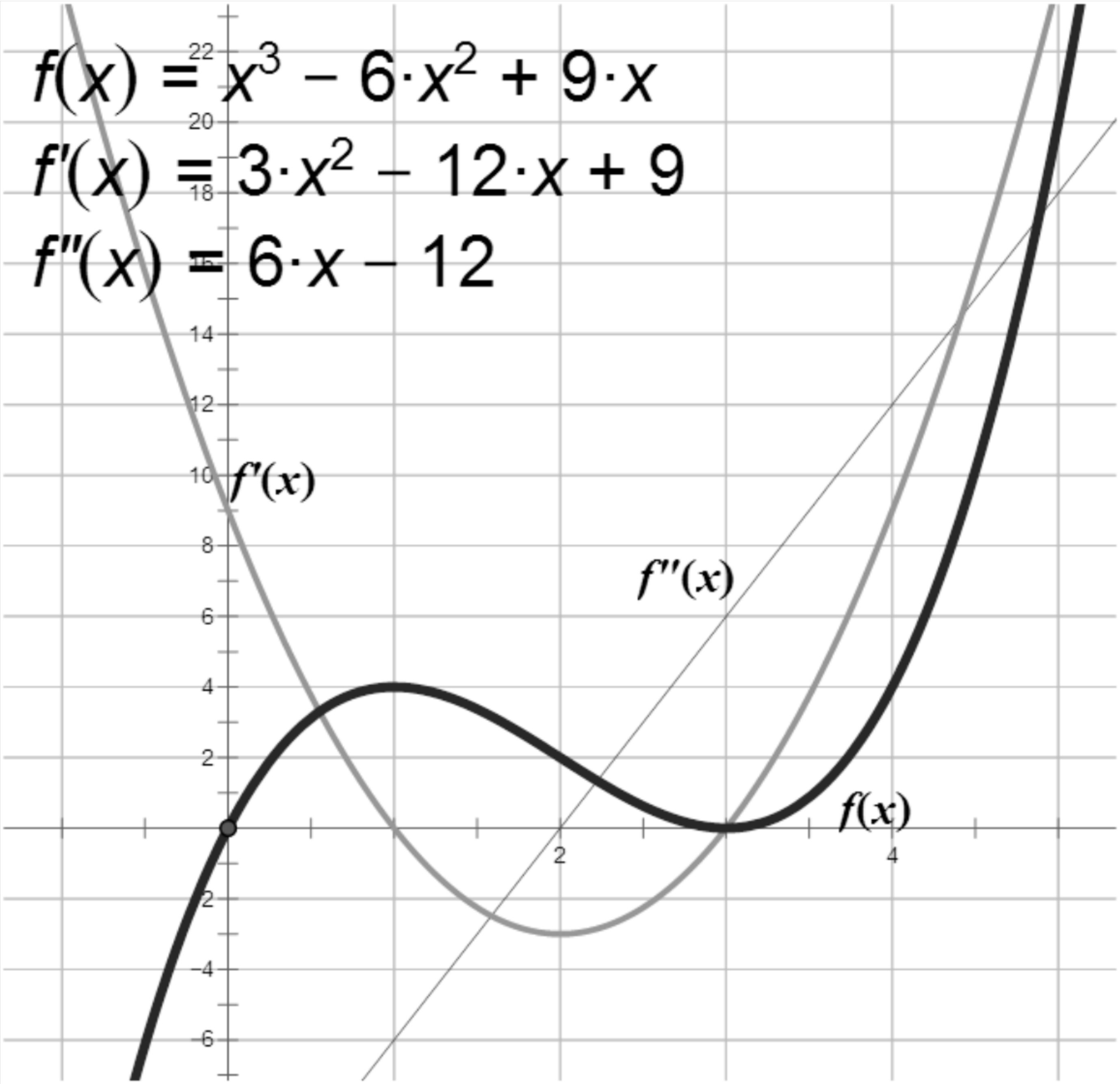


What is the velocity at 2 s? At 4 s?

f ' (2) = -3 ft/sec
f ' (4) = 9 ft/sec

The velocity is at a minimum at t=2, and also negative. This means that the particle is moving to the left.

At t=4, the velocity is positive and increasing, so the particle is moving to the right at an increasing speed.

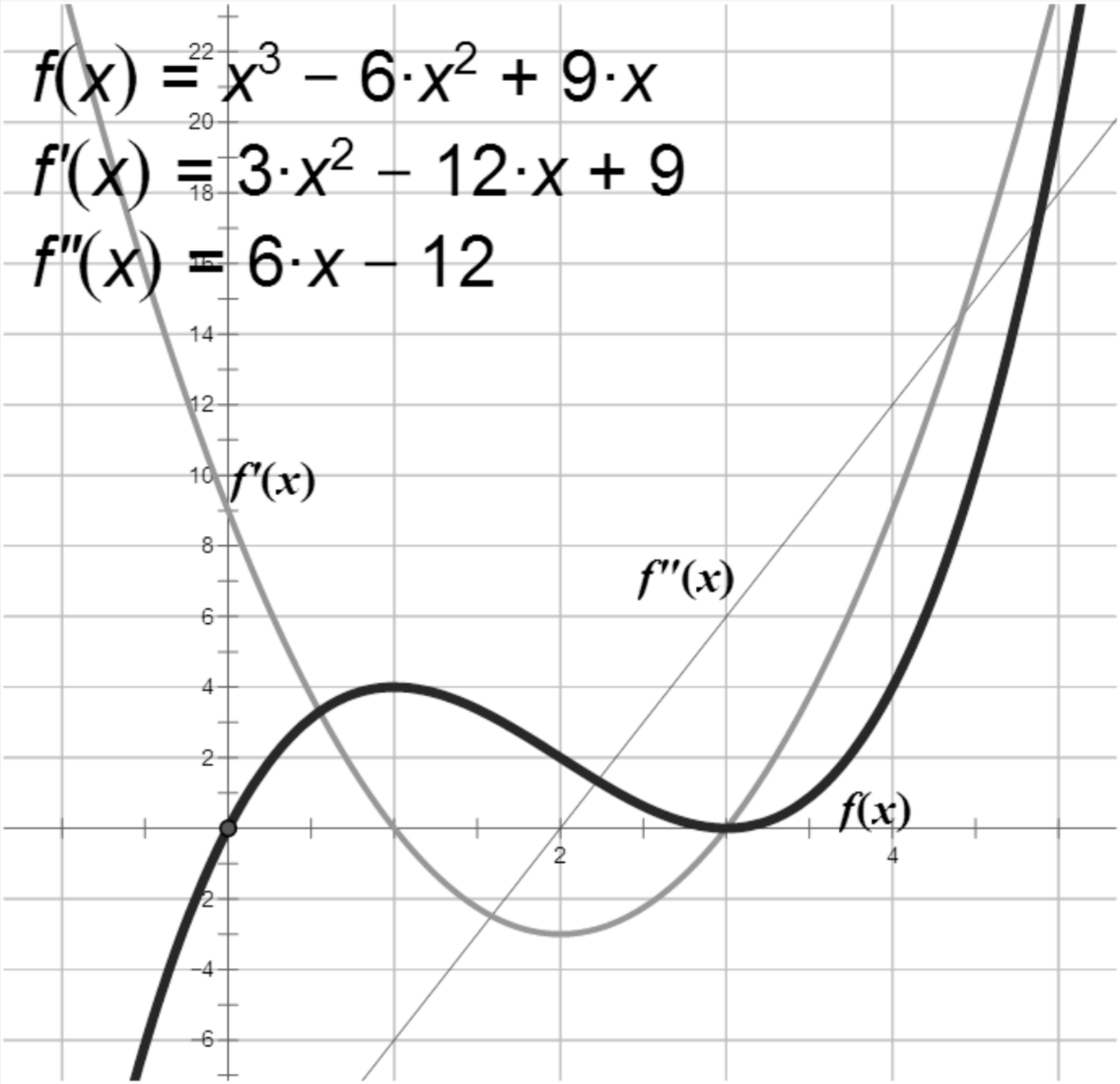


When is the particle at rest?

"at rest" occurs when the velocity is zero.

At t=1 and t=3 the particle is at a max and min respectively.

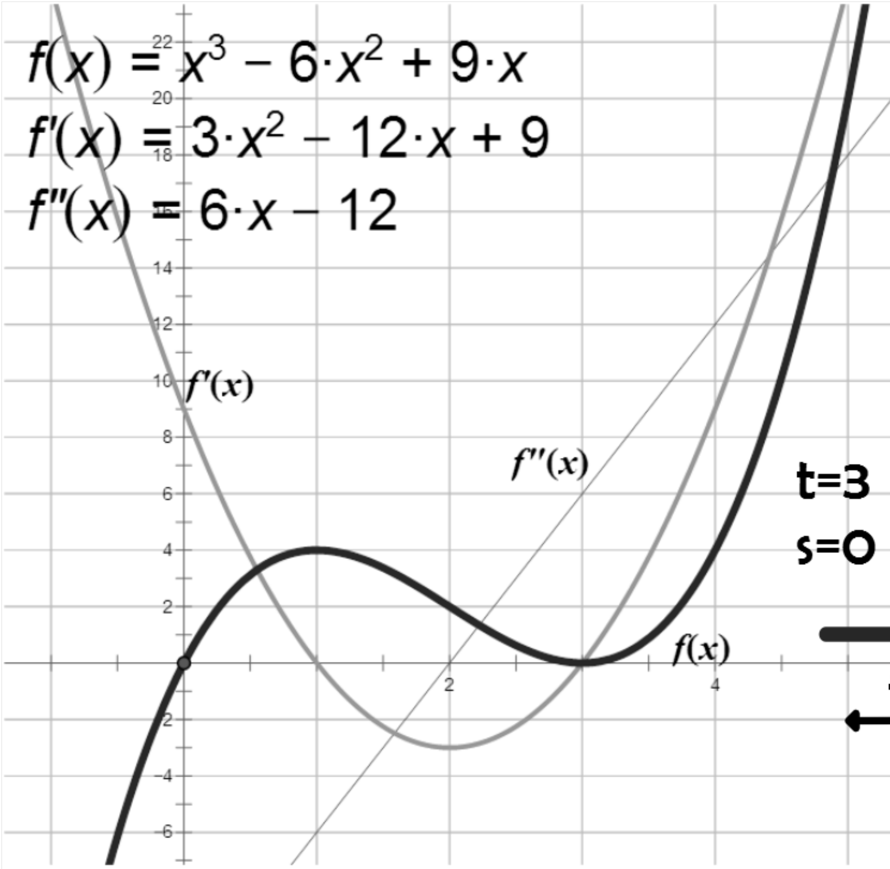
Notice the velocity at these times.

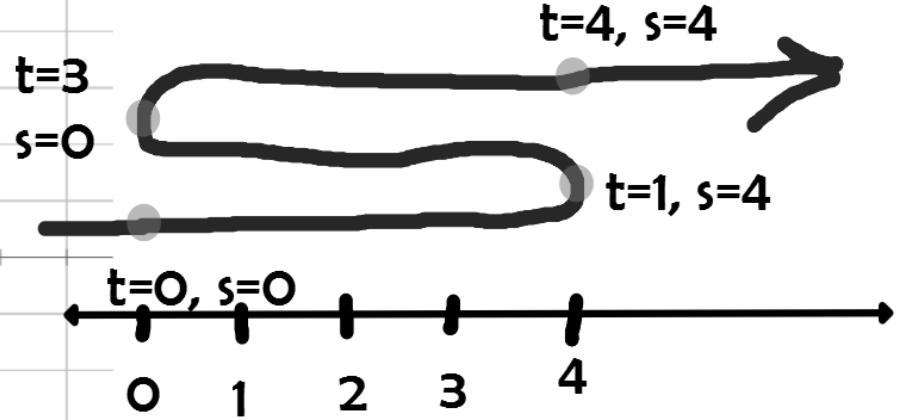


When is the particle moving forward (that is, in the positive direction)?

Moving forward from (0, 1) and (3, 5).
This coincides with when the velocity
is positive.

Draw a diagram to represent the motion of the particle.

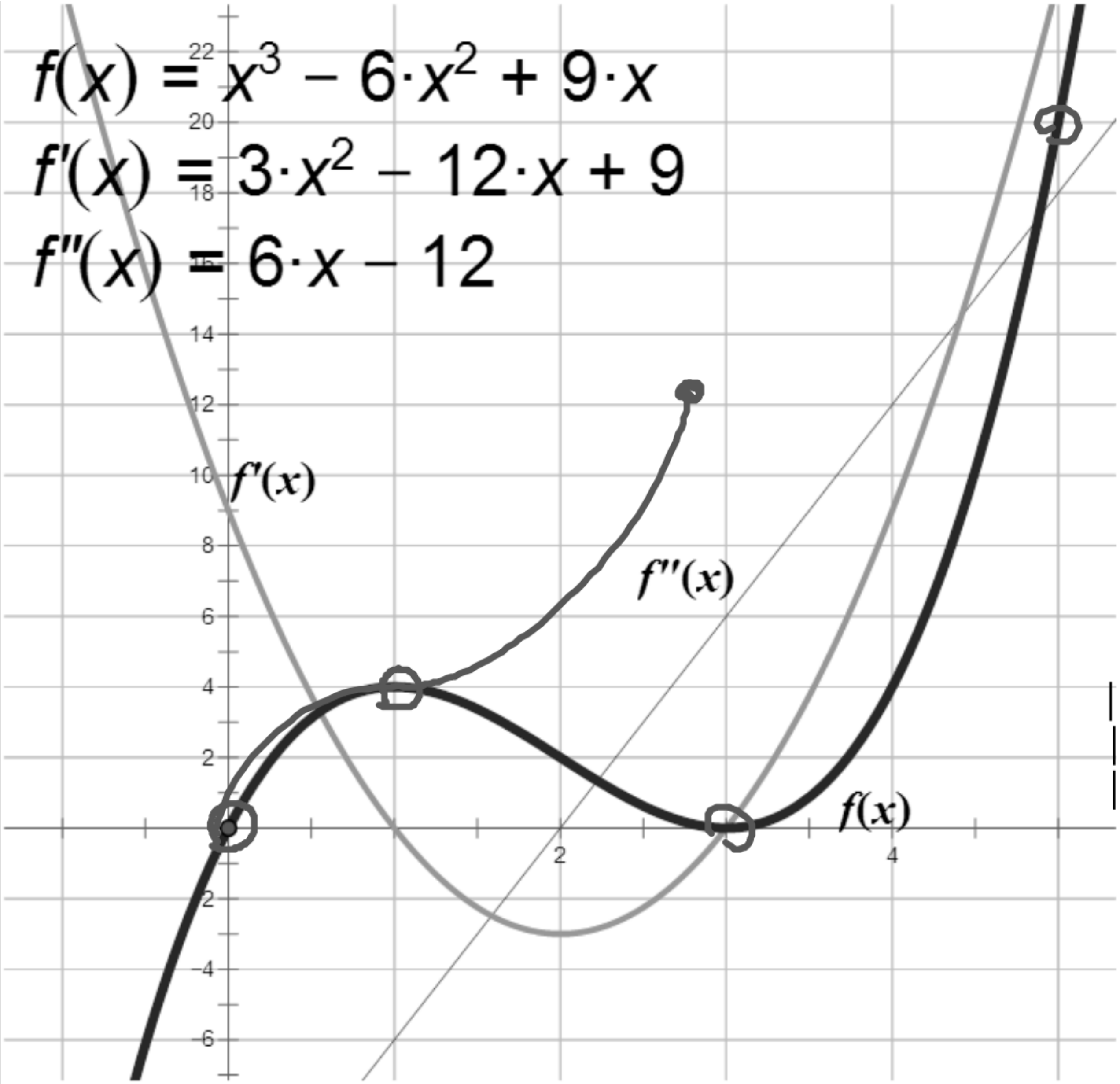




$f(x) = x^3 - 6 \cdot x^2 + 9 \cdot x$

$f'(x) = 3 \cdot x^2 - 12 \cdot x + 9$

$f''(x) = 6 \cdot x - 12$



Find the total distance traveled by the particle during the first five seconds.

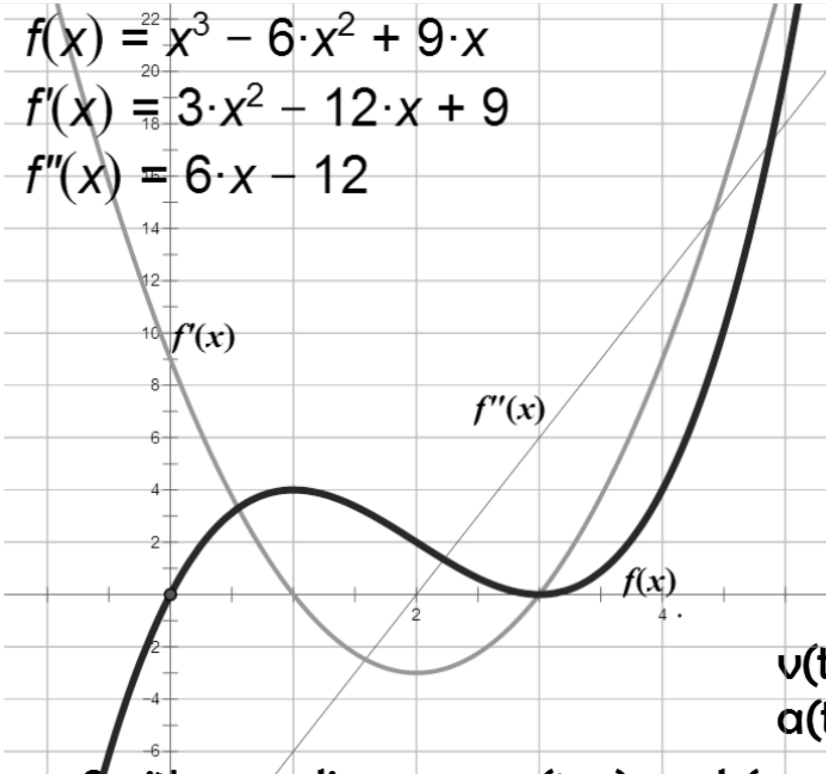
This includes going both forward and backward.

$|f(1) - f(0)| = |4 - 0| = 4$

$|f(3) - f(1)| = |0 - 4| = 4$

$|f(5) - f(3)| = |20 - 0| = 20$

4 + 4 + 20 = 28 feet



When is the particle speeding up? When is it slowing down?

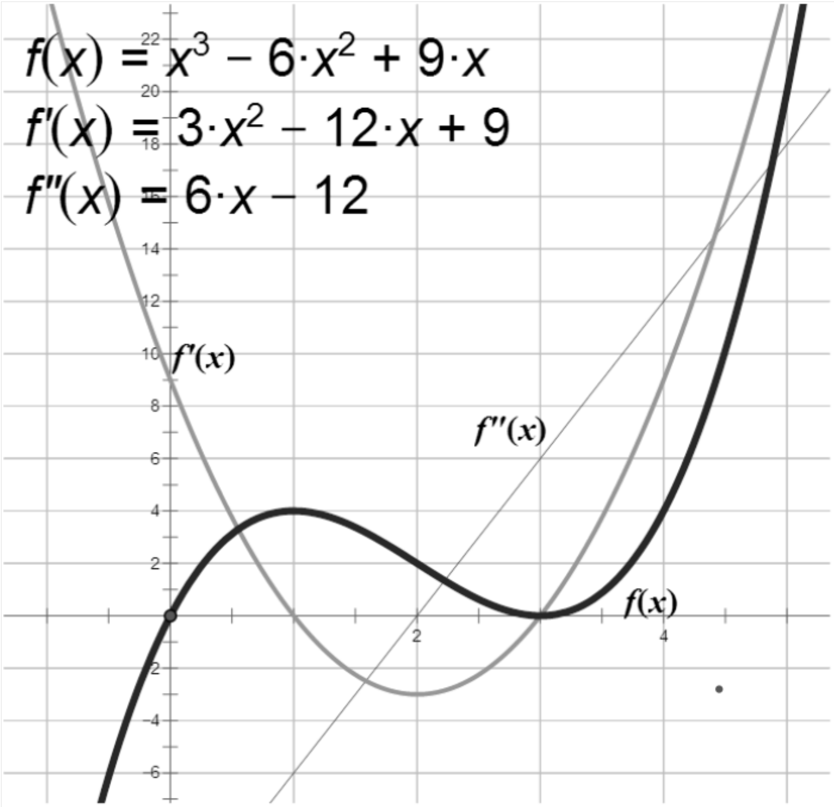
Speeding up occurs when the VELOCITY and ACCELERATION <u>HAVE THE SAME SIGNS</u>.

That is, if velocity is positive and acceleration is positive, it's being "pushed" more positive. The same idea occurs when v(t) and a(t) are negative, but being "pulled" back.

$v(t) > 0$ on (0, 1) and (3, 5) $v(t) < 0$ on (1, 3)
$a(t) > 0$ on (2, 5) $a(t) < 0$ on (0, 2)

So it's speeding up on (1, 2) and (3, 5)
The particle is slowing down when the signs don't match, on (0, 1) and (2, 3)

Mention driving a stick shift and shotgun's got a cup of coffee re: linear acceleration



What is happening to the particle's movement, its velocity, and its acceleration at t = 2 s?

At t=2, s is moving left, v is at a minimum, and a is zero.

The particle is changing from moving left quickly to "not so quickly", velocity is done decreasing and about to increase.

Ex. If a ball is thrown vertically upward with a velocity of 80 ft/s, then it's height after t seconds is $s = 80t - 16t^2$.

a. What is the maximum height reached by the ball?

Here you've got to "get" that at the maximum height the velocity would be zero (it isn't moving for that instant).

$$v(t) = s'(t) = 0 = 80 - 32t$$
$$32t = 80$$
$$t = 2.5\ s$$

So using this t in the position equation:

$$s(2.5) = 80(2.5) - 16(2.5)^2$$
$$s(2.5) = 100\ ft.$$

Ex. If a ball is thrown vertically upward with a velocity of 80 ft/s, then it's height after t seconds is $s = 80t - 16t^2$.

b. What is the velocity of the ball when it is 96 ft above the ground on its way up? On it's way down?

First we need to find WHEN it's at 96 feet.

$$s(t) = 80t - 16t^2 = 96$$
$$0 = 16t^2 - 80t + 96$$
$$0 = 16(t^2 - 5t + 6)$$
$$0 = 16(t - 3)(t - 2)$$

So the ball has a height of 96 feet at $t = 2$ on the way up, and $t = 3$ on the way down.

At these times the velocities are

$$v(2) = 80 - 32(2) = 16 \text{ ft/s}$$
$$v(3) = 80 - 32(3) = -16 \text{ ft/s}$$

Oh, hey! One derivative function you'll need to know for the homework (# 15) that I don't think we've done in class yet...

$$\frac{d}{dx}(a^x) = a^x \ln a$$

Where *a* is a constant