Remember this?

Find the equation of the line tangent to the curve

$k(x) = x^2 + x + 8$ at $x = 5$

use $y - y_1 = m(x - x_1)$

$x_1 = 5$ $y_1 = k(5) = 38$ $m = k'(5)$

$k'(x) = 2x + 1$

$k'(5) = 11$

$y - 38 = 11(x - 5)$ or $y = 11x - 17$

Linear Approximations

Let f be a function with $f(1) = 2$, $f'(1) = 3$, and $f''(1) = 0.0001$

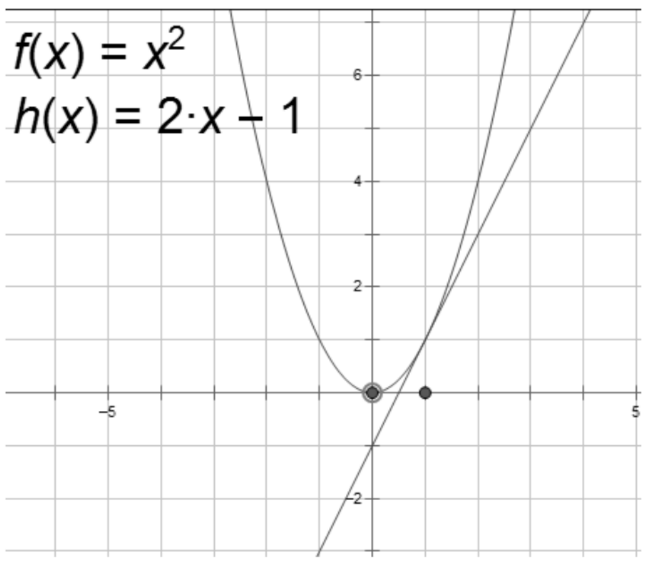
1. Write an equation of the tangent line to the graph of f at $x = 1$, and use it to approximate $f(1.2)$

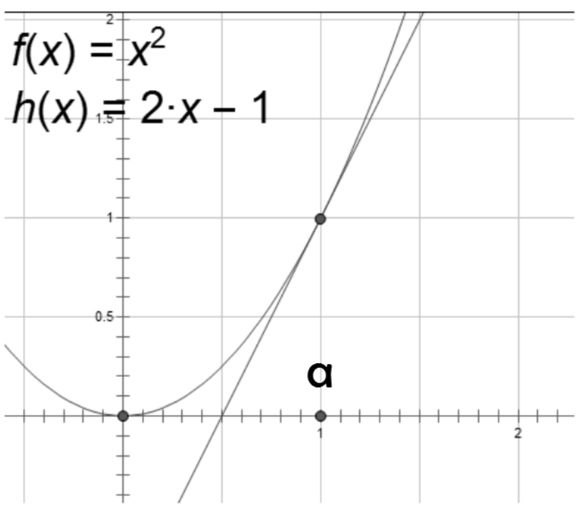
2. Is this a good approximation of $f(1.2)$? Why or why not?

1. $y = 3(x-1) + 2$, $f(1.2) \cong 2.6$

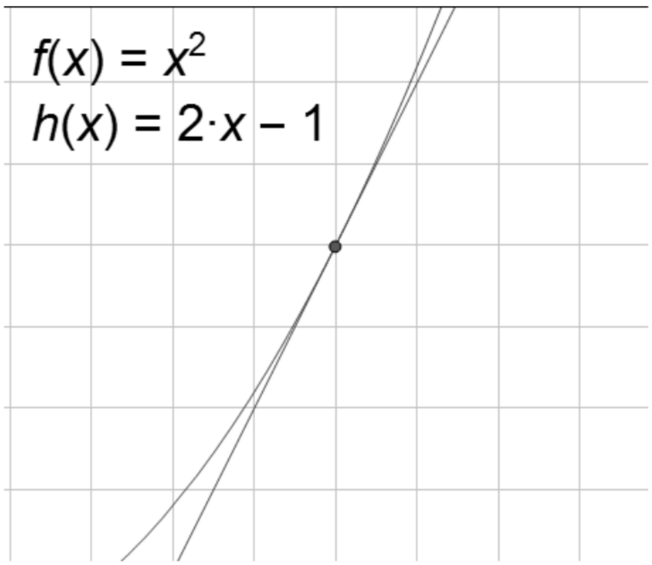
2. Since $f''(1)$ is close to zero, the derivative is changing very little there, meaning the curve approximates a line at $x = 1$; thus, the tangent line approximation is probably pretty good.

A tangent line is the line formed by the points (a, f(a)) and (a+x, f(a+x)) as x approaches a on the function f.
Watch what happens as we zoom in on the point (1,2) when $f(x)=x^2$.

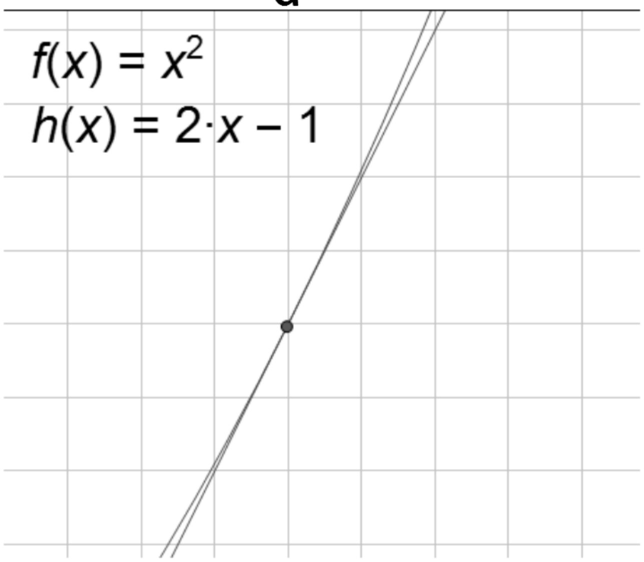



f(x) = x^2
h(x) = 2·x − 1
a
2
1.5
1
0.5
1
2

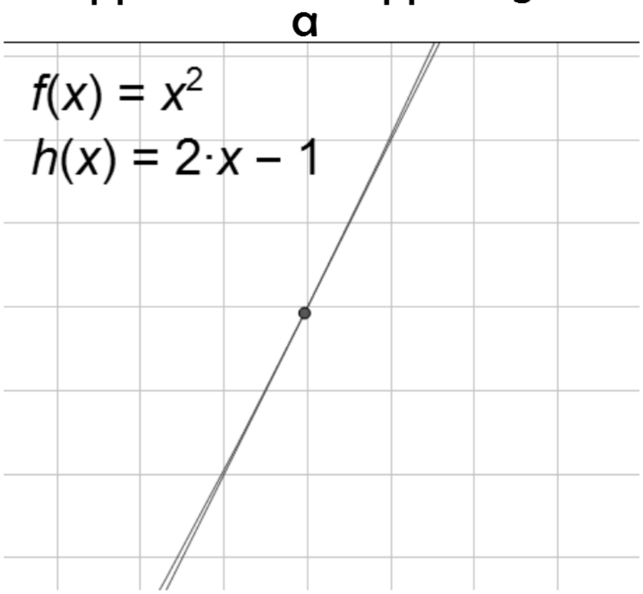
a



a



What appears to be happening?



It seems that the function f at point "a" and the tangent line at point "a" are nearly indistinguishable as we zoom in close to the tangent line.

$$f'\left(a\right) = \frac{f(a) - f(x)}{a - x}$$

Where a and f(a) are known,
x is where we're looking for
the output.

Definition: **Linear Approximation or Tangent Line Approximation**
When the equation of a tangent line is:

$$y = f(a) + f'(a)(x - a)$$

we can use the approximation:

$$f(x) \approx f(a) + f'(a)(x - a)$$

for values very close to "a"

Group work (p.159)

Answers

p.159

1. $f(x) = 1.75(x - 2) + 4$, so $f(1.98) = 3.965$ and $f(2.02) = 4.035$
2. f is concave down, so the approximations are overestimates.
3. The estimates are both 7, because the function is horizontal when $x = 3$.