10.2 - Series

A series is the sum of the terms of a sequence.

A series is also called an "infinite series" - add all the terms of the sequence.

A partial sum is when a finite number of terms of a sequence are added (so, just add up the 8th term, or whatever).

$$s_1 = a_1$$
$$s_2 = a_1 + a_2$$
$$s_3 = a_1 + a_2 + a_3$$

↖ 3rd partial sum

nth partial sum

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

A series is **convergent** if $\lim_{n \to \infty} s_n = s$ exists as a real number.

s is called the sum of the series. Otherwise, the series is **divergent**.

Geometric series

A series is geometric if it can be written as

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \ldots$$

and is <u>convergent</u> if $|r| < 1$ and its sum is $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

If $|r| \geq 1$ the geometric series is <u>divergent</u>.

The partial sum of a geometric series:

$$s_n = \frac{a(1-r^n)}{1-r}$$

Ex. Is the series convergent or divergent?

$$\sum_{n=1}^{\infty} 2^{2n} 3^{1-n}$$

plan: try to rewrite the n^{th} term as ar^{n-1}

$$\sum_{n=1}^{\infty} \left(2^2\right)^n 3^{-(n-1)}$$

$$\sum_{n=1}^{\infty} \frac{(4)^n}{3^{(n-1)}}$$

$$\sum_{n=1}^{\infty} 4\left(\frac{4}{3}\right)^{n-1}$$

Since r = 4/3 > 1 , the series **diverges**.

The <u>harmonic series</u>

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots \quad \text{is } \boxed{\text{divergent}}.$$

If the series $s = \sum_{i=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$

What this is saying:

If a series converges, the values in the sequence must be approaching zero.

n	a_n	s_n
1	3	3
2	0.3	3.3
3	0.03	3.33
4	0.003	3.333
5	0.0003	3.3333

Notice that as the partial sums approach 3 1/3, the sequence a_n approaches zero.

I mean, think about it. If a series converges (so, it stops at a value), the terms being added have to be pretty much nothing.

nth term test (test for divergence)

If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{i=1}^{\infty} a_n$ is divergent.

Ex. Show that the series $\sum_{i=1}^{\infty} \frac{n^2}{5n^2+4}$ diverges.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2}{5n^2+4} = \lim_{n\to\infty} \frac{1}{5+\frac{4}{n^2}} = \frac{1}{5} \neq 0$$

The series diverges by the nth term test.