10.1 Sequences

This section is all about sequences, and specifically whether they converge or diverge.

A sequence has a limit L and we can write

$$\lim_{n \to \infty} a_n = L$$

IF the terms approach L has we make n large enough.

If the limit exists, the sequence is said to converge (is convergent).

If the limit does NOT exist, the sequence diverges (is divergent).

Of course all the old limit laws still apply (look at p. 678) but here are a couple more than might help more than they used to:

Squeeeeze theorem:

If $a_n \le b_n \le c_n$ for $n \ge n_0$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$

This one too:

$$\lim_{n\to\infty} |a_n| = 0, \text{ then } \lim_{n\to\infty} a_n = 0$$

Ex. Determine if the sequence converges:

$$a_n = \frac{(-1)^n}{n}$$

$$\lim_{n\to\infty}\left|\frac{(-1)^n}{n}\right| = \lim_{n\to\infty}\frac{1}{n} = 0$$

The sequence converges toward the number 0.

Look at this:

Does $a_n = \frac{\ln n}{n}$ converge?

$$\lim_{n \to \infty} \frac{\ln n}{n} = \frac{\infty}{\infty} \overset{H}{=} \lim_{n \to \infty} \frac{\frac{1}{n}}{1} = 0$$

yep, don't forget about l'Hospital's rule.

Increasing, decreasing or monotonic?

Monotonic is when the sequence is increasing and decreasing.

Ex. Show that the sequence $a_n = \frac{n}{n+1}$ is decreasing.

Is $\frac{n+1}{(n+1)^2+1} < \frac{n}{n^2+1}$?

$$\frac{n+1}{(n+1)^2+1} < \frac{n}{n^2+1} \Leftrightarrow \left(n^2+1\right)(n+1) < n\left[\,(n+1)^2+1\right]$$

$$\Leftrightarrow n^3+n^2+n+1 < n^3+2n^2+2n$$

$$\Leftrightarrow 1 < n^2+n$$

Since n ≥ 1, we know this is true and the sequence $\{a_n\}$ is decreasing.